744½
~~531.118~~
4
7

METHOD
TO DISCOVER
THE DIFFERENCE
OF THE
EARTH'S DIAMETERS;

PROVING
It's true Ratio to be not leſs variable than as 45 is to
46, and ſhorteſt in it's Pole's Axis 174 Miles.

ELUCIDATED WITH FIVE COPPER PLATES,
With a Variety of
NEW TABLES ſuited to the Subject.

LIKEWISE A METHOD FOR FIXING
AN UNIVERSAL STANDARD
FOR
WEIGHTS and MEASURES.

By THOMAS WILLIAMS.

K

LONDON:
Printed for the AUTHOR, No. 6, *Dowling's Buildings*, near
Gray's Inn Lane; and ſold by J. STOCKDALE,
Piccadilly;
And by Wm. RICHARDSON, Under the *Royal Exchange*.
M:DCC:LXXXVIII.

BRITISH MUSEUM

PREFACE.

IT will be needleſs to endeavour to prove the importance and utility of the knowledge of the true figure of the Earth, in its application to various branches of ſcience. The pains and expence that have been beſtowed to obtain that knowledge, and the reputation of thoſe who have employed their time and abilities for that purpoſe, ſufficiently evince its conſequence; aud render it unneceſſary for me to ſay any thing on that head.

As ſeveral celebrated mathematicians had been ſent to different parts of the Earth, to meaſure diſtant portions of the Arc of the Meridian, I implicitly aſſented to what they laid down; taking it for granted, that their meaſures

ſures and concluſions might be depended upon: nor did I ever ſuſpect the contrary, till about four years ago; when, from a circumſtance of no importance to the public, I graduated the whole meridian, and found a much greater difference in its extremes, than had been given by the above-mentioned obſervers; and that the diameters of the earth muſt vary proportionably with the firſt and laſt degrees.

For ſome time I handed about my thoughts upon this ſubject in manuſcript. Not finding ſo much attention paid to it as I expected, I was induced to print the outlines of it in a pamphlet laſt year: but many perſons who peruſed it, conceiving that the addition of a few plates would greatly illuſtrate what I had advanced; it is now reprinted, and I flatter myſelf, there is no obſtacle to prevent a clear conception of the whole.

I will venture to ſay, that extenſive ſcientific knowledge is by no means requiſite to enable any one to aſcertain this problem: and that a perfect underſtanding of a few plain things, will abundantly ſuffice for that purpoſe.

To exemplify the importance of this ſubject, I will here extract a few paſſages from the Engliſh tranſlation of Maupertuis' book on the figure of the Earth: who was principally concerned in meaſuring the degree in Lapland.

And firſt, preface page 3, he ſays, " This " queſtion, for its curioſity only, might well

" merit the conſideration of philoſophers and " mathematicians: but the advantages ariſing " from the diſcovery of the Earths true figure, " go beyond mere ſpeculation; they are real, " and of very great importance. Were the po- " ſition of places, with reſpect to longitude and " latitude, ever ſo exactly marked on our globes " and charts, it would ſignify little to the find- " ing their true diſtances, while we were igno- " rant of the length of the degrees of the " meredian, and of the parrallels to the Equa- " tor. And if the diſtances of places are not " very well known, to what dangers muſt the " ſhips be expoſed that are bound for them!"

Again, page 11. " It is evident in general, " that Sir Iſaac Newton's figure of a flat Sphe- " roid, and Caſſini's of a long one, will give " very different diſtances of places that have the " ſame longitude and latitude. And it is of " ſome conſequence to navigators not to fancy " they are ſailing on one of thoſe ſpheroids, " while they are really ſailing upon the other. " The miſtake would not be ſo conſiderable, if " the ſhip's courſe lay all in the ſame meridian. " But for places under the ſame parallel, the " difference would be very great. In a courſe " of 100 degrees longitude, there might be a " miſtake of more than two degrees, if ſailing " really upon Sir Iſaac Newton's earth, one " ſhould imagine himſelf to be upon Mr. Caſ-

" ſini's. And how many ſhips have periſhed " by ſmaller miſtakes ?"

" There is this conſideration further: that " till the figure of the Earth is determined, " there is no knowing how far theſe errors may " go.—And in fact it appears, from our mea-" ſures, that ſuch a miſtake will be ſtill great-" er, than from Sir Iſaac Newton's table, it " could be known to be."

" I ſay nothing of the miſtakes which muſt " happen in oblique courſes. It is needleſs to " make any eſtimate of them at preſent: only it " is plain enough, that they would be ſo much " the greater, as the courſe approached to a " paralleliſm with the Equator."

" The errors juſt now mentioned merit cer-" tainly our ſerious attention: and if the ſail-" ors are not at preſent ſenſible how advanta-" geous it would be for them to know the true " figure of the Earth, it is owing rather to the " imperfection than the perfection of their art.

" They are ſubject to a great many other miſ-" takes in the direction of their courſe, their " diſtance run, and the like; amidſt which the " error ariſing from their ignorance of the " Earth's figure lies confounded and hid. Yet " it is ſtill a ſource of error more: and if ever, " (as it is to be hoped) the other elements of " Navigation are brought to perfection, it will " be ſeen of what uſe the exact determination " of the Earth's figure is."

“ This determination would likewiſe be exceedingly
“ uſeful in that important problem,
“ to find the parallelax of the Moon; which
“ would greatly contribute to the compleating
“ a theory of this ſattellite of our Earth; upon
“ which the beſt aſtronomers have always moſt
“ reckoned for the diſcovery of the longitude
“ at ſea.

“ And to come to other objects, lower indeed,
“ but not the leſs uſeful, one may affirm
“ that the perfection of Levelling depends upon
“ the knowledge of the Earth's figure. Such
“ is the chain that connects the ſciences, that
“ the ſame principles which ſerve to direct a
“ ſhip in her courſe, and to trace the Moon in
“ her orbit, ſerves likewiſe to bring water into
“ a fountain or canal.”

“ Page 15. “ Near views of particular advantage,
“ have ſometimes produced great enterprizes
“ for the diſcovery of countries, or of
“ paſſages to abridge certain voyages: but the
“ determination of the figure of the Earth is a
“ general benefit to all nations and ages.”

Page 25, he ſays “ From the increaſe of gravitation,
“ as we found it at the polar circle,
“ this falling in muſt be ſtill greater than Sir
“ Iſaac Newton has made it. And ſome experiments
“ of our academicians at the equator
“ which are lately come to our hands, confirm
“ the ſame thing.” And near the end of the
book, in his reflections on the increaſe of gravitation,

vitation, he obſerves, "The acceleration from
" Paris to Pello is greater than Sir Iſaac New-
" ton's table makes it; and conſequently, ac-
" cording to his theory, the Earth is flatter
" than he determines it to be.—In fine, all the
" experiments which the academicians ſent by
" the king to Peru have made, either at St.
" Domingo or the equator, conſpire with ours
" to make the increaſe of gravitation towards
" the pole, greater than according to Sir Iſaac's
" table, and by conſequence the Earth flatter
" than he has made it."

I ſhall only add here, that I return my ſincere thanks to thoſe friends, who have enabled me to proſecute my deſign. Whatever benefit may ariſe tothe public from my labours, it is entirely owing to their inſtrumentality.

METHOD

OF DISCOVERING

THE TRUE DIFFERENCE

OF THE

DIAMETERS of the EARTH,

&c. &c.

BEFORE I proceed to the proſecution of my ſubject, it may be proper to give the outline of the method I mean to follow. And firſt, in order to obtain the true difference of the diameters, I pay a ſtrict regard to the meaſures of meridional degrees which have been performed in different parts of the Earth; particularly thoſe in the extreme parts in Lapland and Peru; carefully obſerving to increaſe in ſuch an order, from the leſſer extreme to the greater, that ſhall paſs nearly in a mean between all the actually meaſured degrees in the intermediate latitudes. Various methods might be adopted to perform this; but I find none more ſimple or accurate than for the increaſe from the leſſer extreme to the greater, to be proportioned to the natural order of number from one to ninety, anſwerable to the number and order of the degrees from the Equator to the Pole, obſerving to adopt a certain quantity, which I ſhall call the root increment of the Curve, which muſt be ſo much and no more, that when the ſum of the arithmetical progreſſion between the two degrees given is multiplied by it, the difference between thoſe two degrees ſhall be exactly produced.

This root increment when found will graduate every part of the Curve with great exactneſs; the whole length of which will be obtained from the Equator to the Pole. I have clearly ſtated the method of doing this, and from the reſult I gain the real quantity of the pole meridional degree, and find it to be to that at the Equator as 46 to 45. This being evidently demonſtrated, the difference of the diameters is determined by it; which are proved to differ from each other in inverſe proportion, as the firſt and laſt degrees of the meridian, and that as a conſequence, the two extreme degrees at the ſurface, will form equal areas to the centre. And this being ſubſtantiated, will facilitate the gaining another thing, which is, the true ſhape of the Curve itſelf: this is of great conſequence, as it has a ſtrict connection with other ſolutions of much importance.

The method of finding the true Curve is clearly explained, and opens the way to the diſcovery of all the longitudinal degrees in every Latitude; by firſt, transferring the numbers found in the table of meridional degrees in juſt proportion to the Curve line: and afterwards, by having ſome known points firſt given, to graduate the more minute parts by way of tabular calculation: ſimilar to the conſtruction of the table of meridional degrees,

Theſe particulars are cloſely connected with, and dependant on each other; and being united, will obtain the ſolution of the problem in queſtion.

To make way for the firſt thing propoſed, which is, to find the true length of the Earth's Curve; it may be neceſſary to inſert the reſult of the meaſures of various meridional degrees which have already been performed, with what each exceeded that meaſured at the Equator.

This is requiſite in order to know the correſpondence of my generated Curve with thoſe meaſures.

Degrees meaſured.	Latitude.		Toiſes.	Exceeds that at the Equator in Engliſh Ft.
In Peru, on the	D.	M.		
Equator. —	00	00	56753	0000
Cape of G. Hope	33	18	57037	1815
Penſilvania -	39	12	56888	863
Rome } Italy	43	1	56979	1444
Turin	44	44	57138	2461
Hungary - -	45	57	56881	818
France - -	47	00	57060	1962
Vienna - -	48	43	57086	2128
Holland - -	52	4	57145	2505
England - -	52	45	57300	3496
Lapland - -	66	30	57438	4378

By comparing the above ſtatement with my table of meridional degrees and the generated Curve, it will be found, that no meaſured degree varies from the truth, but little more than 700 feet, except that of Hungary; which is upwards of 1000 feet leſs than the true meaſurement.

That at the Cape of Good Hope is near 700 feet too much: that at Penſylvania about the ſame quantity too little. That at Rome about 400 feet too little: that at Turin about 400 feet too much. Thoſe in France, Vienna, and Holland, each, about 200 feet too little. Mr. Norwood's, in England, upwards of 700

too

feet too much. Whilſt thoſe meaſured in Peru and Lapland, at the two greateſt extremes, exactly agree with the tabular numbers.

When it is conſidered what a trifling inaccuracy in meaſuring a degree cauſes an error of many hundred feet, it is not to be wondered at that moſt of the meaſured degrees are not exactly true but rather that they be brought ſo near the truth as they are.

It is alſo obſervable, that the meridional degree meaſured in Lapland (about eight years after Sir Iſaac Newton's deceaſe) was found to be fifty French toiſes, or near 320 Engliſh feet more than Sir Iſaac gave his pole meridional degree itſelf.

And when it is conſidered what a vaſt increaſe there muſt be, in the twenty-two meridional degrees from the polar circle to the pole, it cannot be doubted but that the two extreme degrees of the meridian, muſt differ from each other much more than was thought in Sir Iſaac Newton's time.

It may likewiſe be worthy of remark, that though ſome of the degrees at a great diſtance from the equator, have been made to meaſure leſs than others which were nearer, yet they have all without exception meaſured more than that at the equator.

Thoſe meaſured at Vienna and in Holland exceed the equatorial degree 2000 feet each :

that in England meaſured 3000 feet; and that in Lapland 4400 feet more. An undoubted evidence that the earth is oblated, and ſhorteſt in its axis.

For, as Maupertius obſerves, in his book, page 37th. " If the earth is flatted towards " the poles, a degree of the tereſtrial meridian " will be longer towards the poles than towards " the equator.

This aſſertion of his is an undoubted truth, and on which the certainty of the oblatiſm itſelf depends. For; tho the degree given by the quadrant is not, ſtrictly ſpeaking, proportioned to the curvature on the ſurfaee, as I ſhall prove; yet it is greatly enlarged towards the Poles by the oblatiſm; in conſequence of its falling in a mean proportion between the center which agrees with the curvature, and the common center of gravity; that is by making the two extreme degrees equal to the two diameters of the ſpheroid.

The manner of conſtructing my tables, with the proofs of their veracity, I ſhall deſcribe farther on.

I ſhall therefore proceed to the firſt thing propoſed, which will be fully and clearly exemplified, by Plate 1, and its explanation.

PLATE 1, fig. 1, repreſents the ſection of a quarter of the globe, biſected at right angles through the equator and poles: *a b* is the axis, *c d* the equator; the outſide arch called *quadrant* is 90 equal degrees; the inner line called Earths Curve, ſhews the falling in of the Globe from the quadrant, and is ſuppoſed to be a meridional line, and the real Curve of the Earth. The figures 1, 3, 5, 7, 9, 11, in the inner part of the curve, indicate the order of increaſe, found in the table of meridional degrees; and which agrees beſt with actual meaſurement. The 10th degree from the Equator exceeds that at the Equator by about 100 feet; the 20th exceeds the 10th nearly 300 feet; the 30th the 20th 500 feet, and ſo on throughout the whole curve. If we ſuppoſe the whole number of degrees in the Curve divided into four equal parts, the 23d degree from the equator will meaſure nearly 500 feet more than a degree at the equator. The 45th degree, 1500 feet more than the 23d, and the 67th degree 2500 feet more than the 45th: from which it may be ſafely concluded, that the 90th or pole meridional degree, will meaſure 3500 feet more than the 67th; which is nearly the proportion numbered in the Curve.

The total amount then of all the numbers in the table of meridional degrees, is the true length of the Curve of the Earth, or meridional line from the Equator to the Pole.

The line drawn from the point *f*, which touches near the 60th degree on the Curve, is drawn with two arches of circles, from the centre *e* to the 60th degree, and from thence to the pole, from the centre f; as deſcribed by the rule given for fixing the bounds of the ſurface of the ſphroid.

Fig. 2, plate 1, contains a Curve generated from the exceſſes or unequal quantities found by the table of meridional degrees; or in other words, from the real Curve of the Earth already deſcribed in fig. 1, the perpendicular line A B terminates the whole length of the meridional degree to the left hand of the Equator. The Pole degree will exceed it from B to D or from A to F nearly 8000 feet; which is repreſented by eight equal parts in the ſcale at the bottom. If the diſtance from A to F be taken, and ſet upon the perpendicular line A B, and divided into 90 equal parts, as in the plate; and if what each degree in the table exceeds that at the equator be exactly noted, and each quantity placed at its proper latitude from the perpendicular line, on the 90 equal parts, the exceſſes of the tabular numbers will generate the Curve laid down in the plate; and every 10th degree will fall nearly in the order mark'd on the perpendicular lines which join the generated Curve; and are in the proportion before deſcribed fig. 1.

Again, if the degrees of the perpendicular line be divided into four equal parts, and carried out to touch the generated curve, and a plummet let fall as 1, 3, 5, 7, in the plate, the diſtance of the plumblines will be nearly in the order before deſcribed in fig. 1ſt, from the perpendicular line to the firſt plumbline nearly 500 feet, from 1 to 3, 1500 feet, from 3 to 5, 2500 feet, and from 5 to 7, 3500 feet. This curve then is generated ſimply by the progreſſional increaſe of the numbers up to 90. For if the line A F repreſent the ſum of the progreſſion to 90 (which is equal to 4095) and the height be ſuppoſed the ſame, every number laid down in true proportion will generate one and the ſame curve. The method is as ſimple as it is accurate; it fills up as it were the true quantities of every degree in the curve, when the degree at the equator and one at a diſtant latitude is aſcertained. Now the degree at the equator and that at the polar circle are actually aſcertained; and as far as the polar circle the curve in the plate perfectly agrees with actual meaſurement, for the degree meaſured between TORNEA and KITTIS in LAPLAND, was found to exceed that at the equator nearly 4400 feet, and if this exceſs be taken from the ſcale at bottom and ſet off from the perpendicular line at the proper latitude it will touch the generated curve and perfectly agree with it in the point E and plumbline 5. Of the intermediate degrees which have been meaſured in France, Holland, Penſilvania, and Italy, ſome are more and others leſs than my tabular numbers. The irregularity in the meaſurement of theſe degrees ariſes entirely from accidental circumſtances, as the want of exactneſs in the obſerver, or the vicinity of ſome large mountain, which might attract the plummet a little out of the vertical line. Indeed the ſmalleſt error is productive of great inaccuracy, a deviation from truth of the 1,000th part of an inch on the arc of a ſextant or *quadrant* of 9 feet radius creates an error of 195 feet, is it to be wonderd then that no degree ſhould be exactly meaſured, and is it not more conſiſtent to adopt the method I have laid down in generating my curve? From what I have ſaid, it follows, that if every particular meridional degree be laid down in the order deſcribed to a ſcale, there ariſes a demonſtration, that the 23 degrees from the polar circle to the pole, muſt neceſſarily increaſe more than thoſe from the 45th degree, to the polar circle; becauſe if they be contracted towards the pole, they muſt conſequently paſs off from E towards C in the plate; which is an impoſſability. The generated curve reſpects gravity, and has the ſtricteſt connection with it; therefore the leaſt deviation, either by increaſing or leſſening, argues an abſurd iregularity of the ſurface of the globe; deſtroying its uniform curvature. The unfathomable depths of fluid, in the pacifick and atlantic

Oceans, croſſing the whole globe as it were, ſo many thouſand leagues aſunder; affords another proof that the earth is an uniform and true ſpheroid, and as a conſequence all lands are the ſame. Therefore that irregularity that is found in the intermediate meaſured meridional degrees, ariſing only from accidental circumſtances, no degree towards the pole, can increaſe leſs than the degree preceding it; and conſequently there is a greater increaſe from the 67th degree to the 90th; than there is from the 44th to the 67th.

But as this is of great importance, I will endeavour to illuſtrate it by a remark or two more.

It is obſervable, that at every 10th degree there are two units of additional increaſe: it is manifeſt from this that the common increaſe is leſs in proportion towards the pole, than in the beginning of the curve. For example—2 bears a ſmaller proportion to 15 in 10 degrees towards the pole, than it does to 13 in the 10 preceding degrees: and a ſmaller proportion to 13 in thoſe 10 degrees laſt mentioned, than it does to 11 in the 10 degrees before them: and ſo on to the Equator in the ſame ratio of diminution.

This though apparently inconſiſtent, is nevertheleſs true; and confirms the exiſtence of what I call the root increment; which is a given quantity, eſſentially neceſſary to conſtitute every part throughout the whole Curve: and is rather leſs than two Engliſh feet, viz. 1. 944,—When compared with what is to be added to it near the Equator, this inconſiderable quantity is large; the increaſe being ſo ſmall there: but it becomes very ſmall, when compared with what muſt be added to it, in order to form the proper increment for the degrees near the pole.—This is the reaſon why the generated Curve in the plate is ſo *concave* near the Equator; whilſt near the pole it approaches almoſt to a right line; the effect ariſing only from the diſproportional quantities which the root is added to; for the whole Curve is generated from the ſame root.

For an explanation of my meaning, take an example.—In the table the increment of the 5th degree from the Equater, is leſs than 10 feet; the root 1. 944 being added to make up the quantity 11. 664, which is what the 6th degree exceeds the 5th; whereas at the 60th degree, the quantity is found to be more than 116 feet, the root 1. 944 being added to make up the quantity 118, 584; which is what the 61ſt degree exceeds the 60th, and ſo at any other two extremes in the Curve.

It is demonſtrable and indeed ſelf-evident that the Curve of the Earth being but one ſimple curve, cannot generate two curves: conſequently the table agrees with the whole

Curve as near as may be, this is ſufficiently evinced by the generated Curve laid down on the plate and from what has been ſaid.

The firſt and laſt meridional degrees being proved to be to each other as 46 to 45, the next thing to be known is, what proportion the the diameters of the Earth will bear to that Ratio: or whether it is, or is not the real diameters of the Earth alſo—. This being the ſecond principal point, and of the utmoſt importance to this ſubject, I ſhall lay down ſuch clear and ſatisfactory reaſons, that the diameters of the Earth do differ the one from the other in the proportion of the two extremes, that it ſhall finally appear indiſputable. It may be obſerved that I call the two extreme meridional degrees, degrees of gravity, to diſtinguiſh them from two degrees of curvature at the ſame points: becauſe they widely differ from each other as ſhall be proved hereafter.

That the diameters ſhould be in the above mentioned proportion, ſeems to be natural and almoſt ſelf evident: becauſe this ratio will always form equal areas from the ſurface to the centre of the ſpheroid; or the common centre of gravity. The Planets in their Orbits forming equal areas in equal times, in ſome meaſure confirms this idea; for both reſpecting gravity as the governing power, it is reaſonable to ſuppoſe a correſpondence. To ſet this part of my ſubject in a clear light, I refer to plate 2, which contains five ſections of very flat ſpheroids: namely as 1 to 5 that is 5 in the Equatorial diameter, and 1 in the poles axis: with the diſtribution of the degrees on their ſurfaces; all different from each other.

Had the quadrant been originally divided into 100 equal parts inſtead of 90, and the minutes, ſeconds, and thirds, decimally deduced from that, it would in ſome reſpects have been better for ſhortening calculation than it is at preſent. That the diſtribution of the degrees on the ſurface of a ſphere, ſhould be all perfectly equal, is inconteſtible: but that they ſhould be equal on the ſurface of a Spheroid is abſurd. That they ſhould be ſo conſtructed as that the firſt, and laſt degrees on the ſurface ſhould be in the ſame proportion to each other, as the diameters of the ſpheroid they are upon, that is to ſay, in inverſe proportion, as in fig. 3, is as natural, and conſiſtent as that they ſhould be equal on a ſphere; for both the one, and the other form equal areas to the common centre of gravity.

If the degrees were formed as on the ſurface of the Spheroid fig. the 1ſt. it would be extremely abſurd: and it would be equally ſo if as on fig. 5, where the inequality of the degrees on the ſuperficies, coincides with the curvature found in the two extreme parts of the Spheroid: by which the degrees in the extremes, become eight times as variable as the diameters. Fig. 2, and fig. 4, denote an inferior extreme, for the degrees on fig. 4, are conſtructed to be in a mean between thoſe in fig. 5, and thoſe which are in the true proportion, fig. 3. Equal parts on the ſurface cannot poſſibly exiſt in ſo unequal a body as fig. 2, which is coupled with fig. 4, to denote the inconſiſtency of both. They are however nearer the true mean than, fig. 1, and fig. 5.

Ten degrees in each extreme from the ſurface to the common centre of gravity, are darkened, that the propriety of the true proportion as well as the abſurdity of the extremes might be obvious to ſight.—Proofs will be fully and diſtinctly laid down in the explanation of plate 3, both this plate, and that being adapted to elucidate the ſame thing; the propriety and truth of what is delineated in fig. 3, which is the true mean; and correſponds with the operation of gravity on the ſurface of any proportioned oblate ſpheroid whatever.

Plate 3, In order to prove that what I advance reſpecting the matter illuſtrated by this plate, will not in the leaſt deviate from Sir Iſaac Newton's principles: as gravity reſpects matter,—I ſhall inſert a few paſſages from that great man in his own words. Principia page 42, he ſays " And by this property matter " ſluggiſh, and inactive of itſelf retains all the " power impreſſed upon it. By the degree " of this power of inactivity we primarily

" judge of the quantity of ſolid matter in each " body. We conclude it to be a property " eſſential to all matter. Page 43, the degree " of this power of inactivity is in every body " proportioned to the quantity of ſolid matter " in it. We can with certainty compare together the degrees of this power of inactivity in different bodies, particularly this power is proportioned to the weight of bodies. " Page 60, that its force upon any body does " not at all depend upon the ſhape of the body; but that it continues conſtantly the " ſame, without any variation in the ſame " body, whatever change be made in the figure of the body. This has given reaſon to " conclude that the power of gravity acts upon " bodies in proportion to the quantity of matter in them. Page 248, that the force of " the ſame attracting body is exerted on others " exactly in proportion to the quantity of matter in the body attracted. Page 252, that " this attraction extends itſelf to every particle " of matter in the attracted body, and that no " portion of matter whatever is exempted from " the influence. Page 299, that the form of " the ſolid parts of the Earth makes no alteration in the figure the water will take, I ſay, " this figure of the Earth is the ſame as it " would receive were it entirely a Globe of " Water, provided that Water were of the " ſame denſity as the ſubſtance of the Globe."

I have

Plate 1.

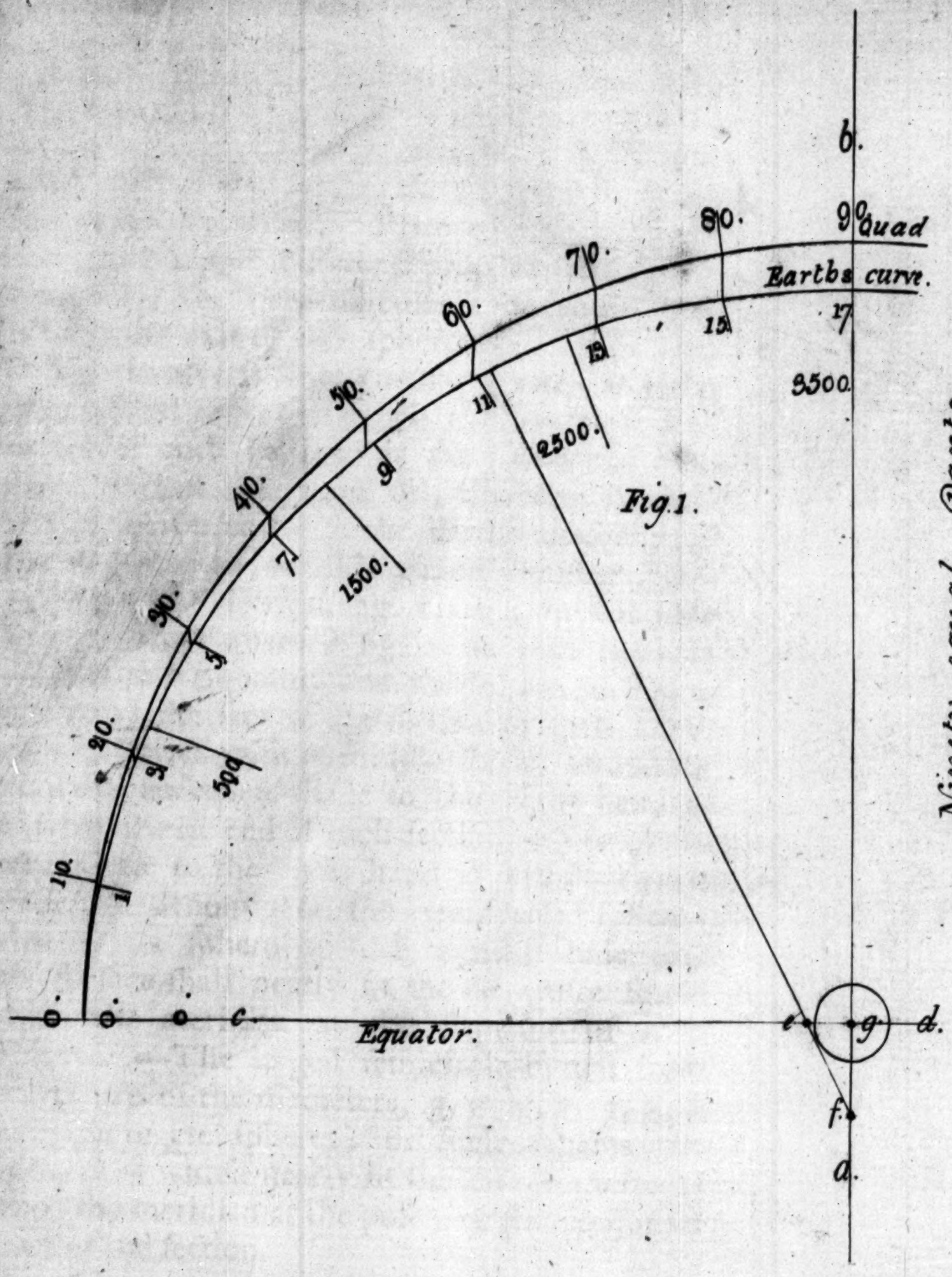


Fig. 1.

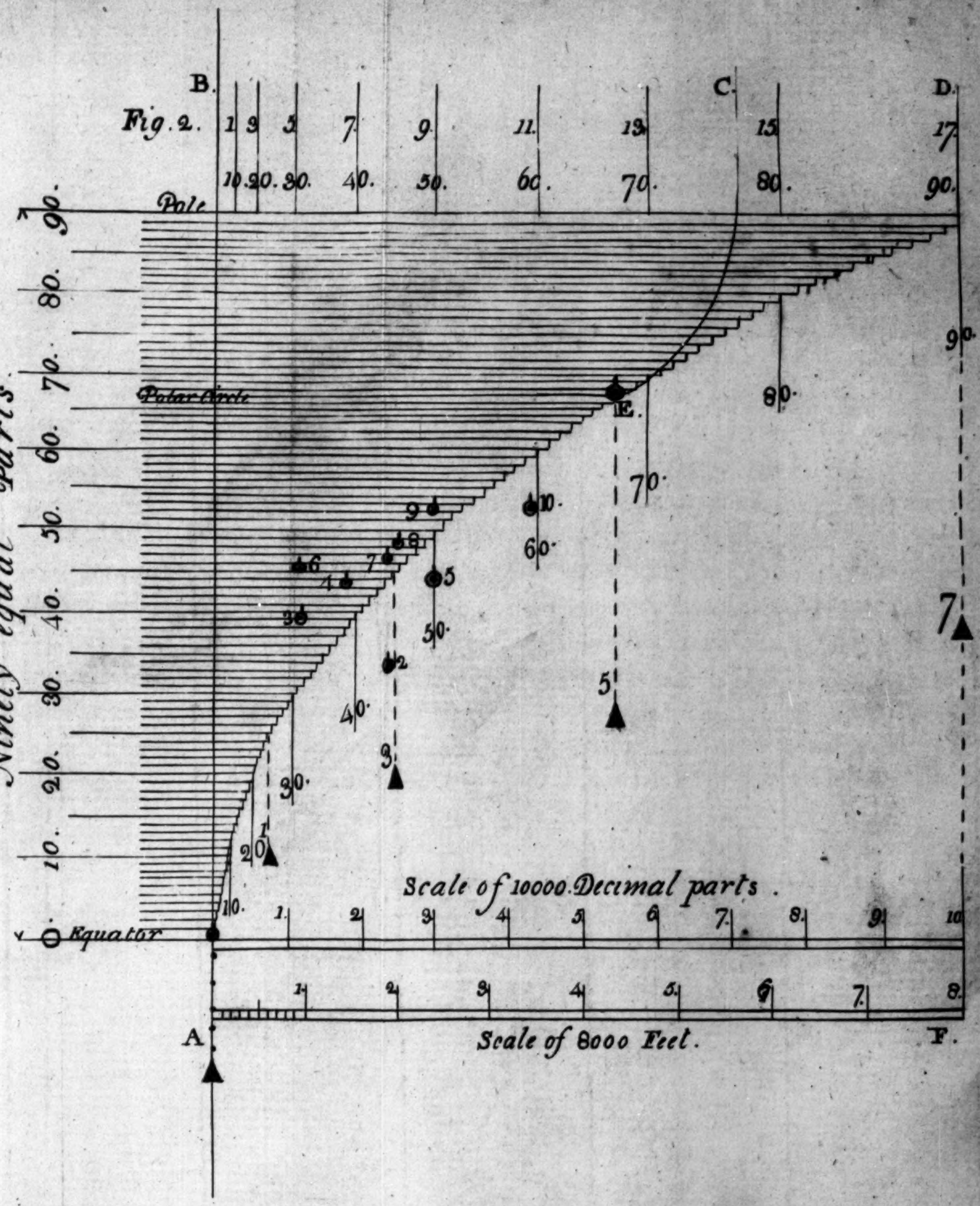


Fig. 2.

I have ſubjoined theſe quotations that thoſe who read the definition of this plate may have a rule at hand to judge whether any thing I aſſert ſhall in the leaſt contradict the doctrine laid down by them. Plate 3, then contains three ſections of different proportioned oblate ſpheroids; the perpendicular and central line is the poles axis of each ſpheroid.

The tranſverſe lines cutting the axis at right angles, and paſſing through the tranſverſe diameter of each ſection, is the equatorial diameter of each. Suppoſe this diameter in each ſection to be equal to the earths diameter at the Equator or to this round number 8000 miles;—The ſection, fig. 1, is 4 in the axis to 5 on the Equator? fig. 2, is 1 in the axis to 2 on the Equator, and the ſection at fig. 3, as 1 to 5, the face of the ſection of each ſpheroid is ſhaded with a middle ſhade, including alſo the darkened ſhade to the right hand at the trranſverſe end of each ſection. The darkened circle to the right hand at the tranſverſe end. I denominate the tranſverſe ſphere; that is, a ſphere of ſuch a magnitude that its ſurface ſhall nearly fit the curvature found upon the meridian at the Equator of each ſpheroid.—The largeſt ſemicircle drawn from the centre of the diameters, g, g. g, g. g, g. I call conjugate ſpheres; or ſuch ſpheres the ſurfaces of which nearly fit the curvature found upon the meridian at the pole; or the conjugate part of the ſection.

The circle, lightly ſhaded, drawn to touch the tranſverſe of each ſection, I name a circumſcribing ſphere, that is, a ſphere of equal diameter with the tranſverſe of each Spheroid: I alſo conſider it as a perfect ſphere equal to what each ſpheroid would be if not oblated. In a line with the tranſverſe diameter of each ſection are other circles drawn with a light ſhade as at a b, c d, e f, the circles to the left hand, a, c, e, are anſwerable to ſuch ſpheres as are attached to gravity in the conjugate part of each ſpheroid, and are in contradiſtinction to the conjugate ſpheres, g g, g g, g g, which reſpect the pole curvature.

The circles b, d, f, are anſwerable to ſpheres which reſpect gravity in the tranſverſe of each ſpheroid; in contradiſtinction to the darkened tranſverſe ſpheres which reſpect the tranſverſe Curvature. The darkened ſphere then, and the conjugate ſpheres are in ſuch extremes the one from the other, conſidered as ſpheres as diameters or radii proportioned to the curvature connected with the ſurface in the extreme parts of each ſpheroid. The ſpheres alſo which reſpect gravity in the extremes of each ſpheroid are in the ſame proportion as the two diameters of each; for the ſphere of gravity *b*, is four fifths of the diameter of the ſphere of gravity, a, equal to the diameter of the ſection fig. 1, to which they belong. Again the ſphere *d*, is half that of the ſphere, *c*, equal to the diameters of the ſection fig. 2. The ſphere *f*, is one fifth of the diameter of the ſphere *e*, equal to the diameters of the ſection fig. 3. Between the centre of the tranſverſe darkened ſphere, and the centre of the ſection, or the common centre of gravity, and on the tranſverſe diameter are other centres marked, denoting the points or length of radii from the tranſverſe ſurface, which are anſwerable to the degree reſpecting gravity on the tranſverſe of each ſpheroid. Alſo on the perpendicular line or axis is marked a centre, falling in a mean proportion between the centre of the conjugate ſphere, and the centre of the ſection; or common centre of gravity. From the mean centre on the tranſverſe to that on the axis are dotted lines which are nearly in the order of the length of radii, correſponding to the degrees of gravity on the ſurface.—When drawn at right angles from the tangents at the ſurface, the dotted line directs itſelf nearly parrellel to the axis from the firſt part of the degrees at the Equator, and the remainder of the ninety degrees paſſing in the direction laid down, meets the axis nearly the ſame as the angles marked on the plate. The firſt of the mean proportiouals on the tranſverſe compared with the laſt on the axis falls in the ſame proportion as the reſpective diameters of each ſection: alſo from the centre of the darkened tranſverſe ſphere to the centre of the conjugate ſphere are dotted lines, denoting the line in which the radii to the degrees reſpecting curvature would move, were it poſſible that degrees of gravity could take place according to curvature.

That the degrees meaſured by the quadrant cannot originate from this, but muſt neceſſarily be attached to the mean proportionals, which are alſo connected with the proportion of the diameters is what is to be demonſtrated on the principles as gravity reſpects matter.

The firſt thing I would notice reſpecting theſe three flat ſpheroids is this; in fig. 1, the diameters ſtand in the proportion of 4 to 5, and the tranſverſe ſphere compared with the conjugate ſphere or what I call the ratio of curvature, is as 1 to 2, nearly. So in fig. 2, the diameters ſtand in the proportion of 1 to 2, and the tranſverſe compared with the conjugate ſphere is nearly as 1 to 7.—On fig. 3, while the diameters ſtand as 1 to 5, the tranſverſe ſphere compared with the conjugate ſtands in the proportion of 1 to 40; which is more than twice the difference of the diameters in fig 1; fourtimes the difference of thoſe in fig. 2; and nearly eight-times as great as the difference of the diameters in fig. 3. As the ratio of curvature then flies off from the ratio of the diameters, in theſe flat ſpheroids in this unequal and extravagant manner, it is inconſiſtent with reaſon to ſuppoſe it can exiſt in connection with theſe variable proportions: for gravity muſt be more fixed, and proportioned to the matter contained in each ſpheroid.

The point of attraction cannot poſſibly exiſt at the common centre of gravity as in fig. 1, becauſe part of the matter of the circumſcribing ſphere towards the conjugate is taken away neither can it exiſt at the centre of the darkened ſphere; by reaſon of the quantity of matter yet contained in the other part of the ſpheroid: it is conſequently attached to a certain mean between the two.

By making the firſt and laſt degrees equal, to the two diameters the degree at the tranſverſe anſwers to a ſphere of the magnitude of *b*, which is a mean between the tranſverſe, and circumſcribing ſpheres; and is reaſonable and conſiſtent—. Alſo at the conjugate or pole, a degree will be formed anſwerable to a ſphere between the conjugate and circumſcribing ſpheres.

That the point of attraction cannot be at the common centre of gravity is evident; becauſe the quantity of matter round the pole is greatly increaſed by being oblated: neither can it exiſt at the centre of the conjugate ſphere *g. g*, which fits the ſurface at the pole, becauſe the conjugate ſphere contains ſuch an immenſe quantity of matter more than the oblate ſpheroid itſelf, which muſt of courſe be attracted to a mean as marked on the plate which anſwers to a ſphere of the magnitude of *a*, larger indeed than the circumſcribing ſphere, but far inferior to the conjugate ſphere *g. g.* This is perfectly conſonant, as gravity reſpects matter.

The demonſtration by fig. 2, is as rational in every point of view: for though the quantity of matter in the ſpheroid given is much leſs than in fig. 1, yet the tranſverſe ſphere to fit the curvature upon the meridian at its Equator, becomes far leſs in proportion to the ſpheroid itſelf; which is the true reaſon why the radius anſwerable to the degree which reſpects gravity at the tranſverſe part of the ſpheroid, ſo greatly exceeds the radius of the darkened tranverſe ſphere in fig. 1; though much farther than that from the common centre of gravity: for it is here found to be as much again, and anſwers to the gravity ſphere *d*. So alſo at the pole or conjugate of fig. 2, the length of radius or point of attraction falls at the centre marked in the plate, betwen the common centre of gravity, and the centre of the conjugate ſphere, and is anſwerable to the magnitude of the conjugate gravity ſphere *e*; which indeed exceeds the circumſcribing ſphere more than in fig. 1, but is however leſs in proportion to the conjugate ſphere *g.g*, fig. 2: and by making the firſt and laſt degrees equal to the two diameters which are as 1 to 2, the ſpheres of gravity *c*, and *d*, are in the ſame proportion as thoſe of the ſpheroid given, and is juſt in every reſpect.

Fig. 3, is carried to a yet greater extreme with regard to its oblatiſm, but by making the firſt and laſt degrees of gravity to exiſt on its ſurface, and to be in the ſame proportion as its two diameters, it is perfectly conſiſtent, and agreeable to reaſon. In the firſt place the tranſverſe ſphere is ſo very ſmall in proportion to the quantity of matter in the ſpheroid given, that it is inconceivable that gravity can poſſibly be connected with it. The darkened tranſverſe ſphere is but as 1 to 400, when compared with the ſolid contents of the ſpheroid, and the ſolid contents of the ſpheroid given being compared with the conjugate ſphere nearly as 1 to 400 alſo: now 400 × 400 = 160000 conſequently the darkened tranſverſe ſphere compared with the conjugate ſphere *g. g*, is more variable than as 100,000 to 1, though the proportion of the ſpheroid given is but as 5 to 1; by which it is fully evident, that the degrees reſpecting gravity cannot poſſibly exiſt with the curvature, but are really in the proportion of the diameters. The following conſiderations will ſtill further manifeſt this.

Suppoſe the tranſverſe of the Spheroid 8000 miles in diameter. The tranſverſe ſphere being ſo very ſmall in proportion to the whole quantity of matter in the ſpheroid given, the radius anſwerable to the degree of gravity at the equator falls at the mean point, marked in the plate, and agrees with the gravity ſphere *f*, which is about three times as great as the radius anſwerable to the tranſverſe ſphere: this proportion will be found to be perfectly conſiſtent as gravity reſpects matter, for the point of attraction will neceſſarily fall much further in than the darkened tranſverſe ſphere, becauſe of the quantity of matter in the ſpheroid given and it will alſo be removed further off from the common centre of gravity than fig. 2, becauſe of the great diſtance off the tranſverſe ſurface from the common centre of gravity in proportion to the quantity of matter contained in the ſpheroid it being ſo exceedingly flat.

And with reſpect to the pole or conjugate part as the radius anſwerable to the degree reſpecting gravity, cannot be ſuppoſed to exiſt in the centre of the ſpheroid the common centre of gravity; ſo it would be an equal abſurdity to ſuppoſe that it can exiſt at the centre of the conjugate ſphere *g*. *g*, becauſe it contains 400 times the quantity of matter of the ſpheroid given: it will therefore neceſſarily be attracted to a point between the two, which is marked on the plate and is anſwerable to the magnitude of the gravity ſphere *e*, and exceeds the circumſcribing ſphere more than in fig. 1, or fig. 2, but yet is leſs in proportion to the conjugate ſphere *g*. *g*, fig. 3, than either fig. 1, or fig. 2.

But it may be aſked, how comes it to paſs if gravity is ſo cloſely connected with matter that the gravity ſphere anſwerable to the pole

Plate 2

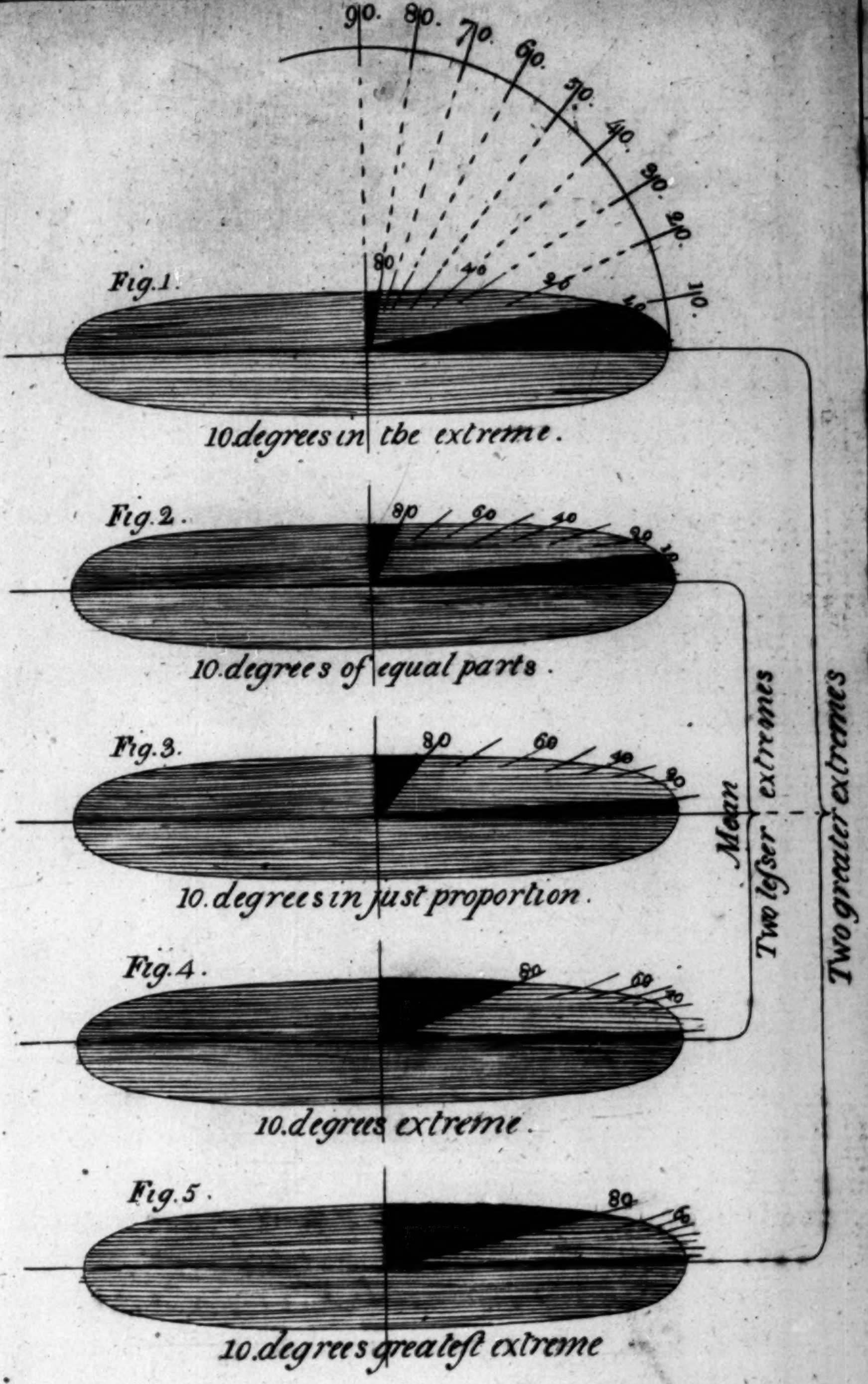


Fig. 1.

10. degrees in the extreme.

Fig. 2.

10. degrees of equal parts.

Fig. 3.

10. degrees in just proportion.

Fig. 4.

10. degrees extreme.

Fig. 5.

10. degrees greatest extreme

Plate 3

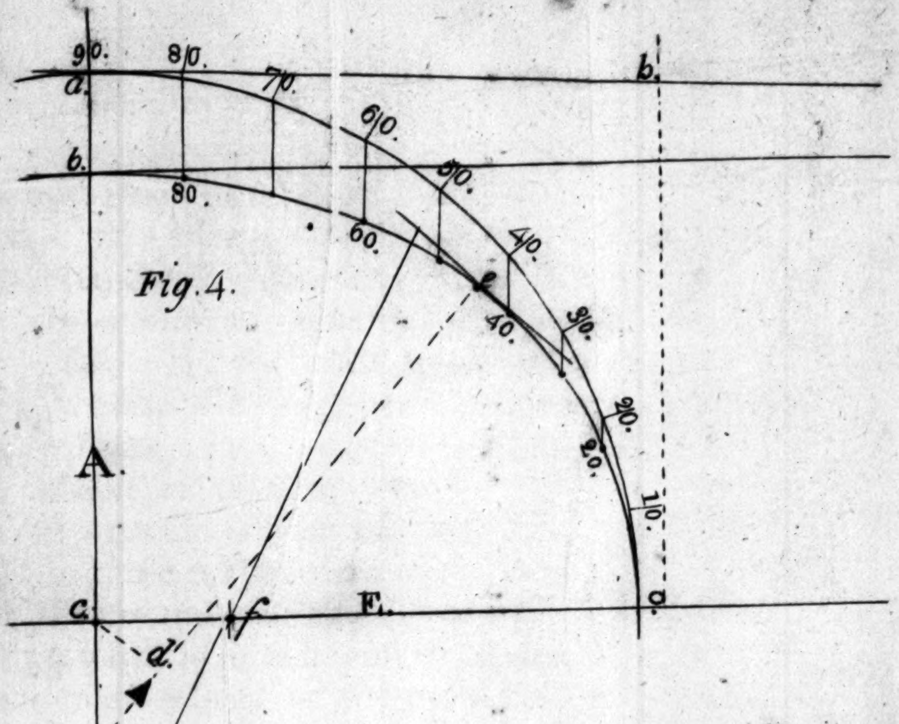


Fig. 4.

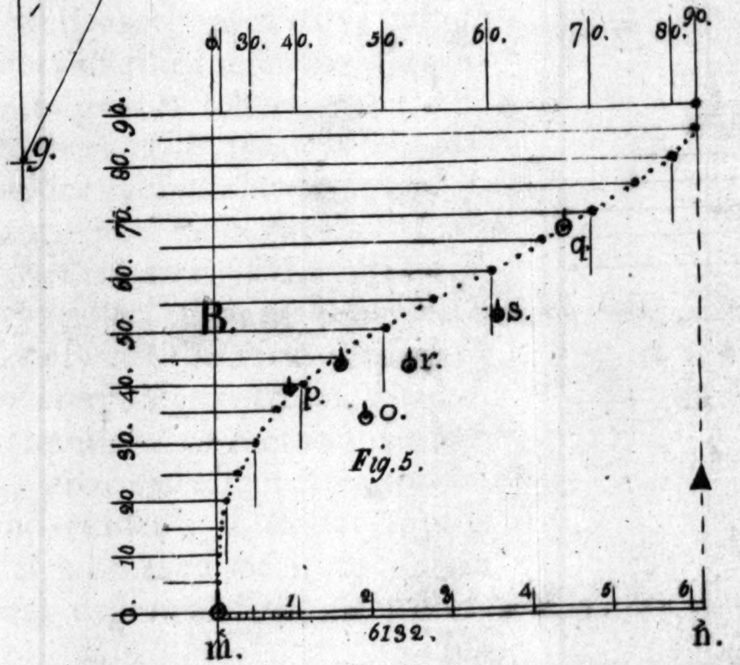


Fig. 5.

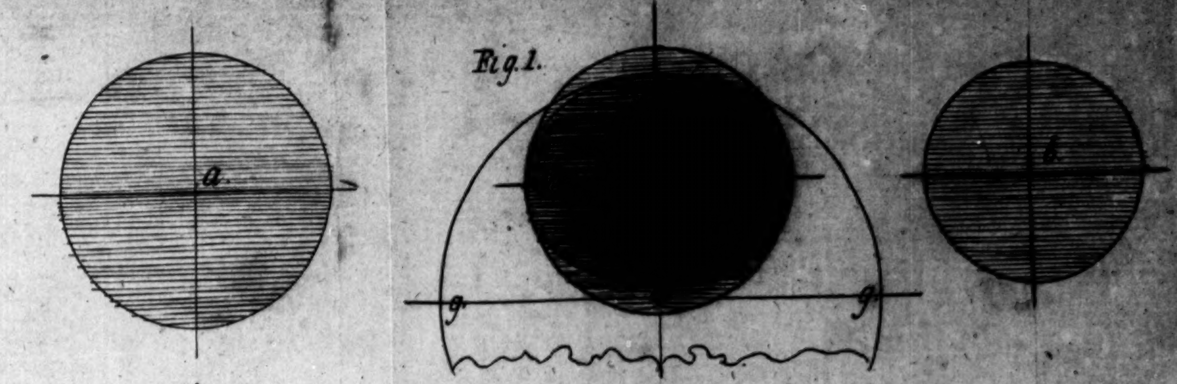


Fig. 1.

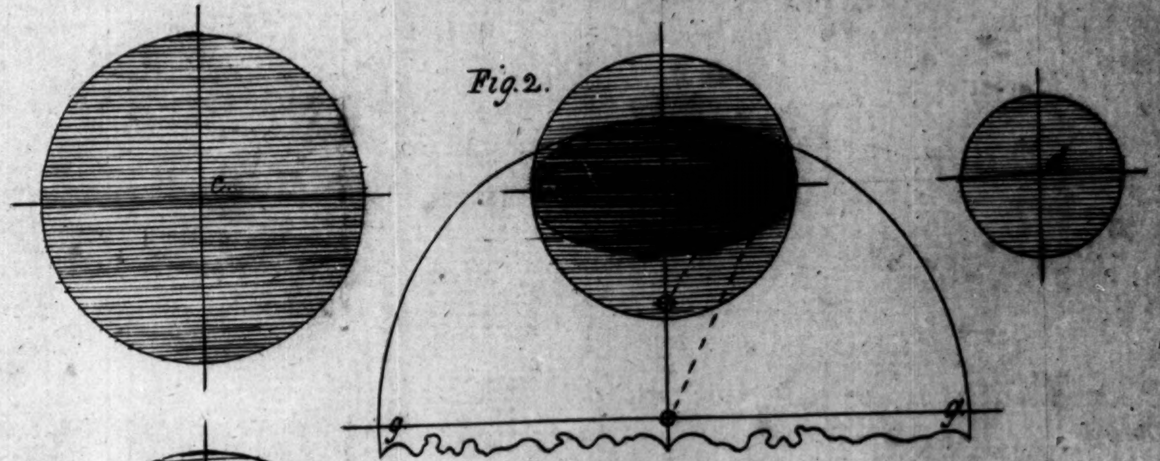


Fig. 2.

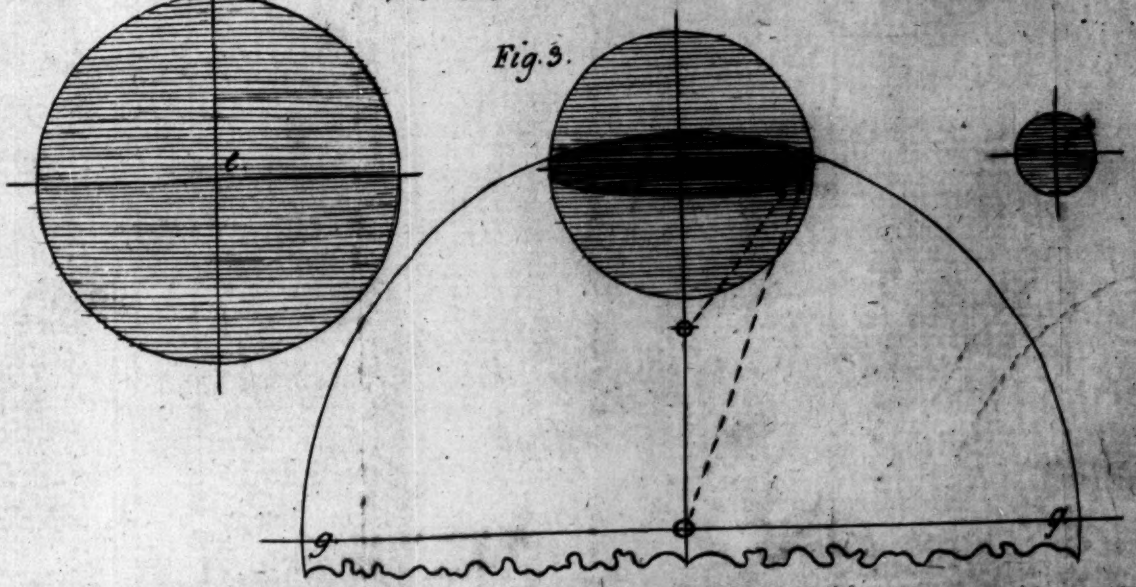


Fig. 3.

degree at the conjugate always increaſes, while the quantity of matter in the ſpheroid given diminiſhes? I anſwer, that the more the ſpheroid is oblated, the greater the quantity of matter is increaſed near the polar part of it in proportion to the tranſverſe diameter, which is evident from the conjugate ſpheres of each ſection: that in fig. 3, is ſuppoſed to fit the pole ſurface, of courſe a ſection made at the pole of fig. 3, will contain a great deal more matter than a ſection of the ſame depth from the ſurface of fig. 1: but either of theſe ſections will contain more matter than one of the ſame depth on the circumſcribing ſphere. Alſo gravity acting with ſo much power on the ſurface has particular reſpect to the proximity of the matter as well as the quantity, which ſupports the propriety of the increaſing magnitude of the conjugate gravity ſphere.

From what has been ſaid then it appears evidently, that gravity cannot be in the proportion of curvature but muſt inevitably be governed by the two diameters of the ſpheroid; and in the two extremes always becomes a mean proportional between the ratio reſpecting curvature, and the common centre of gravity. It is of little importance what method is taken to prove any thing, provided the matter treated of appear clear and evident, I flatter myſelf that the mode I have purſued both to obtain the pole meridional degree and the determining the diameters from the two extreme degrees, however it may be regarded by the learned world, is certain and juſt, and will be ſatisfactory to every impartial perſon.

Plate 4, fig. 1, contains curves drawn in a variety of proportions within the quadrant of 90 equal degrees. The proportion of each curve is inſerted, as 1 to 5, 2 to 5, 3 to 5, 4 to 5, 9 to 10, &c. Likewiſe on each of them is laid down a proportional diſtribution of the 90 unequal degrees; nearly in the proper order in which they fall when transferred from their reſpective curve tables, that is, the firſt and laſt degrees are in inverſe proportion to the two diameters, ſo that, in each extreme they will form equal areas to the centre. Every tenth degree, marked on each curve, is ſuppoſed to form an equal area to the centre at *o*; and to fall at the points where each tenth degree of gravity really is on the ſurface of the ſpheroid.

Wherever the 45th degree falls on the ſurface of either of theſe curves, the curvature or convexity at that point will be equal to that of the quadrant or great circle, which circumſcribes each ſpheroid. This circumſtance helps to confirm the evidence before given reſpecting this matter, for as the tranſverſe part of the ninety unequal degrees on each curve muſt neceſſarily anſwer to a circle leſs in diameter than one anſwerable to the quadrant which circumſcribes the curve, ſo alſo the conjugate part of the 90 degrees will neceſſarily exceed the diameter of a circle correſponding with the circumſcribing quadrant. Nothing can be more reaſonable than that half the 90 degrees ſhould fall only on that part of the curve which agrees moſt with the circumſcribing circle. But were the degrees on the ſurface of the curve to be diſtributed in the proportion which will neareſt fit all parts of the curve, this circumſtance never would take place; but on the contrary, at the point where the 45th degree falls, they would fly off towards the tranſverſe part of the curve, and in flat curves ſo prodigiouſly, that it is evidently an abſurdity to ſuppoſe that the 45th degree, which reſpects gravity, can exiſt at any ſuch point in the curve.

A line drawn from the point in the curve, where the 45th degree reſpecting gravity falls to the centre at *o*; will divide the face of the ſection into two equal parts.

That the two extremes will form equal areas to the centre, *o*, may be eaſily known, for the firſt and laſt degrees in the curve tables are exactly in inverſe proportion with the diameters of the ſpheroid they belong to, and to which they are transferred, for example, in the curve in the proportion of 1 to 5, 5 at the ſurface multiplied into 1 radius at the conjugate is equal to 1, at the ſurface multiplied into 5 radius at the tranſverſe.

It is the ſame in any proportioned curve whatever. So in the ſection of the earth, 46, radius from the centre to 45, on the ſurface at the equator, is an equal area with 45 radius from the centre to 46, on the ſurface at the pole.

I have before endeavoured to prove that gravity is connected with the diameters, and it is very certain by the diſtribution of the degrees on the ſurfaces of the curves in this plate that the extreme points muſt form equal areas to the common centre. Whether the middle 10 degrees in theſe flat curves do ſo in the ſtricteſt ſenſe, or whether there be a little dificiency, is of no great importance, they being uſed only to elucidate the idea. An inconſiderable error ſuppoſing there to be one on ſuch very flat curves as theſe in the plate, vaniſhes, and becomes of no conſequence when applied to the real curve of the earth which comes nearer to the quadrant than 29 to 30.

The advantages ariſing from this mode of diſtributing the degrees of gravity in curve tables, and transferring them in juſt proportion to their reſpective curves, are very great. Where the degree of gravity on the curve is equal to the degree on the quadrant or circumſcribing circle is diſcovered by it exactly.

Were the ſurface of the earth a perfect elipſis, by knowing that the diameters are equal to the firſt and laſt degrees of that elipſis, the length of the circumſcribing quadrant can be known to a great nicety.—My tables of the lengths of any elipſis to the quadrant, having the diameters given, will ſerve for this purpoſe. In table 4 againſt 45 to 46 diameters, ſtands 71, 1572, for the length of the elipſis to 72, the quadrant. Theſe would be the proportions to work by, were the ſurface an elipſis. But it is not found to be ſo. How much it deviates from one may be known by the following method.

On a large ſcale of 8 feet 4 inches radius, lay down a true quadrant by ſetting off 6 feet, 8 feet, and the diagonal of 10 feet, to be the right angle; and as we have proved that the diameters are as 45 to 46 ſet in the 46th part, equal to 2, 1, inches for the falling in of the pole, and draw with the eliptical compaſſes an elipſis repreſenting the ſurface of the earth. Alſo nicely divide the quadrant into 90 degrees, and from every 10th degree at leaſt; draw ordinates parallel to the axis: the places where theſe ordinates will touch the elipſis will be nearly the points in which the degrees on the earths ſurface will ſtand.

It muſt be conſidered that there is nearly 8000 feet difference between the pole meridional degree and that at the equator, and that both form equal areas to the centre of the quadrant. next therefore, divide the oblatiſm of the diagram 2, 1 inches, into 8 eqaul parts, and obſerve in the table of meridional degrees the particular degrees which exceed that on the equator through the whole curve. At the 32nd. degree will be found one of thoſe parts, or 1000 feet, to be ſet in from the quadrant: at the 44th degree, two ſuch parts, and ſo in the ſame manner through the whole curve; then by dividing each of theſe parts into 10, and by obſerving in the table where every 100 feet increaſes, ſet in from the quadrant for every 100 feet, one of theſe ſubdiviſions; and by proceeding through the whole curve in this manner its true formation will be gained.

At the 45th degree it will be found that this curve riſes above the elipſis, about 20 miles, or half an inch, according to the ſcale of the diagram; which ſhews, that the greateſt prominency is between the 50th and 60th degrees; the 60th degree being about 25 miles above the elipſis.

But this may be reſolved with great eaſe by figures. Suppoſe it were required to be known how much the 52nd degree on which London ſtands, falls in from the great circle. Subſtract the degree at the equator from the 52nd, there will remain 2679 feet: then ſay, if 8000 feet difference at 90, give 87 miles from the great circle or quadrant, what will 2679 feet give? the reſult will be 29, 1 miles. By this means the true ſhape of the curve is gained with the greateſt accuracy.

The real quantity of the longitudinal degree on the equator can be diſcovered alſo by this as near as may be wanted; and with much more truth than it can be meaſured, ſuppoſing the degrees of latitude be firſt given true. I will give one example at length. Suppoſe it be required what degree of gravity on the curve in the proportion of 1 to 5, is equal to the degree on the quadrant. The total of all the numbers in the curve table as 1 to 5, is the length of the curve line, as 1 to 5, in the plate. The proportional length of the curve line to the quadrant which circumſcribe it may be accounted for in the manner following. Firſt draw the diagram to a large ſcale, and divide the quadrant into 90 degrees, then with the compaſſes unmoved, nicely obſerve, how many ſuch chords or parts of chords, are found in the curve line. Then ſay. As the number of the diviſions, and parts on the curve, is to the 90 diviſions on the quadrant, ſo is the total of the numbers in the table, to a certain number which will repreſent the length of the quadrant: this number being divided by 90 will give the proportional quantity of a ſingle degree of the quadrant; and if done right will be found equal to the 71ſt degree in this flat curve —Proceed in the ſame manner with each curve in the plate, and it will be found that the quadrant degree will fall nearly in the following order.—That on 1 to 5, at the 71ſt degree.

gree. That on 2 to 5, at the 75th. That on 3 to 5, at the 78th: That on 4 to 5, at the 81ſt, and that 9 to 10, at the 81ſt, and nearly an half.

The above proceſs if applied to the real curve of the earth will give a diameter at the equator of 8010 miles. This method of proceeding if performed with truth, will give the longitudinal degree at the equator much more exact than it can be meaſured by the ableſt Mathematicians, conſidering the diſadvantages they are ſubject to; as the fixing a meridian, the equation of time, the earths motion on its axis being not leſs than 1600 feet in a ſecond of time: theſe things being conſidered, it is very clear, that it is almoſt impoſſible to meaſure a longitudinal degree on the equator within 3 or 400 feet, while by the method above deſcribed it may be obtained within 100 feet

Fig. 2, is a ſcale of the proportion in which the radii belonging to the degrees which reſpect gravity and curvature paſs by the centre of the earth. Suppoſing, *a*, the centre, the radii reſpecting the degrees of gravity paſs from *d*, at the equator, by *b* to *g* on the axis, and the length of radii proportioned to curvature paſs from *h* on the equator by *c* to *i* on the axis. The number of miles in the extreme parts are marked in the plate.

Fig.

Plate. 4.

Degrees reſpecting Gravity

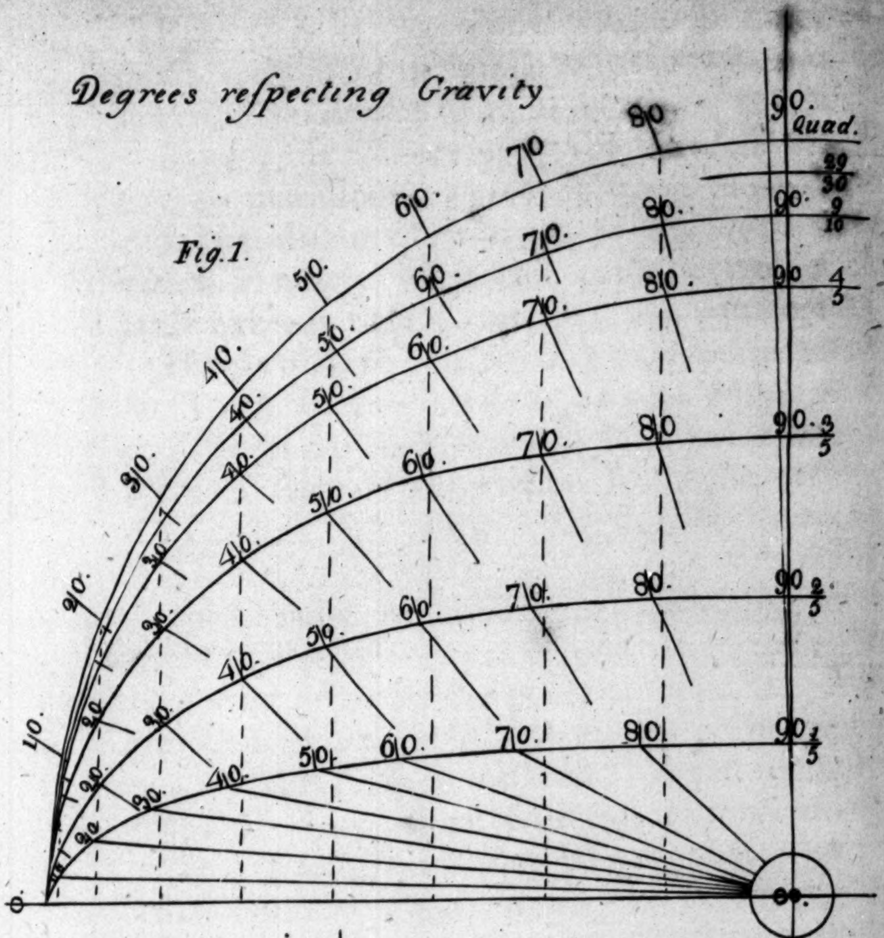


Fig. 1.

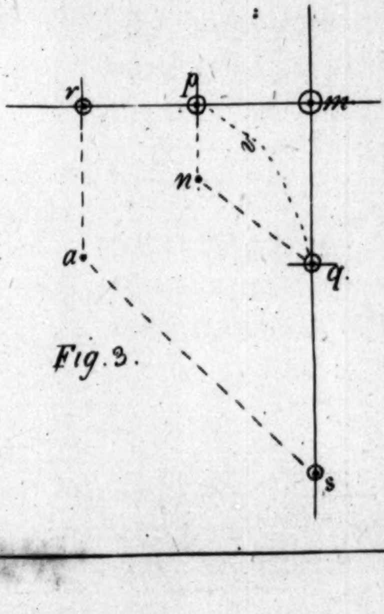


Fig. 3.

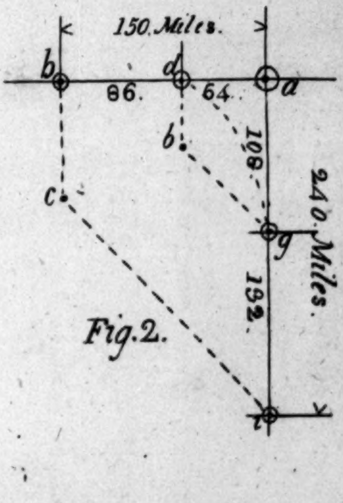


Fig. 2.

Fig. 3, is of the ſame nature in every reſpect, only it correſponds with a ſpheroid in the proportion of 4 to 5, as fig. 1.

Plate 5, is to exemplify the progreſs of the firſt 40 meridional degrees in my table—I will call the diſtance from A to F, 1600 feet, which is nearly what the 40th degree in the table exceeds that meaſured at the equator.

If this quantity be ſet up on the perpendicular line A B, and divided into 20 equal parts, and the true quantity of every ſecond degree ſet off to a ſcale of proportion from the perpendicular line A B, they will generate the curve laid down in the plate.

In the ſame manner if what the degree meaſured in Penſilvania exceeded that at the equator be ſet off from the perpendicular line at its proper latitude it will fall at the point near *b*, and by the ſame proceſs, that meaſured at the Cape of Good Hope will fall near *a*, in the plate. From this it may be obſerved, that the tabular numbers paſs in a mean between both, and as a whole curve or part of a curve cannot generate two curves, it is impoſſible it can deviate from that order, and paſs off from the plumb line, 5 up towards C.

Again, if every tenth degree from the equator *o* to 40, on the plate, be carried out from the perpendicular line, A B, the 10th degree will meet the plumb line, 1, the 20th 3, the 30th 5, and the 40th 7. And this circumſtance attends any proportional quantity of degrees whatever which may be given; They will generate the ſame kind of curve, whether 20 degrees only be taken or 40, 60, 70 or 80 degrees.

This is a full confirmation of the proofs laid down in the explanation of plate 1, fig. 1, (which takes in the whole curve) that the pole meridional degree is actually obtained by the table.

The proportional ſcales at bottom have reſpect to the tables of the diameters, and curvature of various proportioned ſpheroids, which will be referred to in the proper place.

Explanatiou of the method of conſtructing the Tables.

HAVING in the foregoing remarks frequently referred to the Tables found at the end of this tract, it will be proper to give here a particular explanation of the manner by which they are conſtructed; in order to enable any perſon to examine whether I have formed them with truth, or whether that mode of calculation be competent for what it is employed to prove.

I ſhall begin with the table of meridional degrees, and lay down the way to find what I call the root increment. This is done as follows. The quantity of 2 meridional degrees muſt firſt be given, ſuppoſe the 54th in England at 365680, 840 feet, and the 44th degree in France, or Italy at 364718, 560 feet:—firſt it is required to be known what is the root increment, and then what is the quantity of the pole meridional degree and the equatorial degree by that increment.

Subſtract the leſſer degree given from the greater, and make the difference the dividend: next find the ſum of the progreſſion between the degrees meaſured, and make this a diviſor: the quotient is the root increment. Example.

	Feet.	
The 54th degree is as above	365680	840
The 44th degree - -	364718	560
Difference	962	280

Next multiply 49 5 by 10—49. 5 × 10 = 495 the ſum of the progreſſion between. Then 495) 962, 280 (1 944 quotient which is the root increment required. The ſum of the progreſſion to the 44th degree is equal to 990, which multiplied by 1 944—gives 1924, 56: this product ſubtracted from the 44th degree the remainder is 362, 794 feet, for the meridional degree at the equator. Again, the ſum of the progreſſion of 90 being 4095, multiply that

by

by the root 1 944, and the product will be 7960 68; which being added to the equatorial degree, gives the pole meridional degree 370754 68 feet, as in the table. I have introduced the decimal parts in this example, to be ſtrictly conformable to the table, and I give the preference to this increment, becauſe it will admit repeated diviſions by 60 without falling into ſurd quantities; which enables me to generate all the unequal minutes, and ſeconds of the degrees perfectly correct, without any remainder.

The root increment, 1 944 being thus found, it muſt be continually added to itſelf from the beginning to the 90th degree, as may be ſeen in the table: this fixes a juſt arrangement of the increments, and the amount at any particular degree being added to the degree at the equator, the true quantity of that degree will be found:—as may likewiſe be ſeen by referring to the table.

Again, ſuppoſe the 54th degree were given at 365636 feet, and the 44th at 364646 feet. The difference would be 990 feet; the ſum of the progreſſion between 495; the root exactly 2 feet; the equatorial degree 128 feet leſs than in the table, the pole degree 102 feet more; the ratio nearly 45 to 44, and the axis 178 miles ſhorteſt.

From the former root increment 1 944 the quantity of all the variable minutes, and ſeconds may be found by dividing it repeatedly by 60, which diſcovers a mean increaſe. If the increaſe of the degree upon the firſt ten minutes from the equator be required, the proceſs will ſtand thus.

	Root	Mean	Additional	½ Additional
60.)	1.944	.324	.00054	.00027

From the mean exceſs, .0324 take half the additional, .00027 remains the 30th term, .03213—from which take 29 additionals, and there will remain .01647.

This laſt remainder will be a beginning quantity for the increaſe of the degree upon the firſt minute from the equator, and by the continual addition of the increment for the minutes .00054 a ſeries of increaſing quantities will be produced, which added to the degree given will bring the true quantity of every minute of that degree: as follows.

	Increments.	Feet. Parts.
	.00000	362794.00000
1	.01647	362794.01647
	.01701	362794.03348
	.01755	362794.05103
	.01809	362794.06912
5	.01863	362794.08775
	.01917	362794.10692
	.01971	362794.12663
	.02025	362794.14688
	.02079	362794.16767
10	.02133	362794.18900
Additional	.00054	

This is the real increaſe of the firſt 10 minutes of the firſt meridional degree from the equator, and the ſame proceſs being carried on, the increment of every degree through the whole curve will be exactly touched. The increaſe of each ſecond may be diſcovered by the ſame method: they will of courſe agree with the minutes as exactly as the minutes with the degrees.

A continual diviſion by 60, produces as follows,

	Feet.	Parts.
Increment for the degrees	1	.944
Mean		.0324
Increment for the minutes		.00054
Mean		.000009
Increment for the ſeconds		.00000015

Note, as 60 is an even number the mean always falls at 30.5.

The next tables I will explain, are thoſe reſpecting the ratio of the curvature found on the two extreme parts of any ſpheroid, having its diameters firſt given.

In plate 4, are a variety of eliptic curves of different proportions as 1 to 5, 2 to 5, 3 to 5, &c, which have been already treated of.

Nothing is more certain than that the radius correſponding with a circle, which will neareſt fit the tranſverſe part of the curve, in the proportion of 1 to 5, will be continually in-

increaſing

creaſing towards the tranſverſe parts of the curves as 2 to 5, 3 to 5, 4 to 5, and 9 to 10, till it attains the length of the radius of the quadrant or great circle, which circumſcribes each elipſis. And it is equally certain that the radius to a circle which will neareſt fit the conjugate part of each curve, will increaſe from the radius of the circumſcribing circle, till it fits the curves 9 to 10, 4 to 5, 3 to 5, 2 to 5, and comes back to the conjugate part of the elipſis it proceeded from.

Now if a certain number be given for the length of radius belonging to the quadrant, and which I will fix at 120, and another number in the ſame proportion to this number, as the radius of a circle anſwerable to the tranſverſe part of the curve, 1 to 5, is to the radius of the quadrant, which number I fix at 8, an increaſing quantity will be found paſſing from 8 to 120, which in its progreſs will nearly fit the curvature in the tranſverſe part of each elipſis, and alſo increaſing in the ſame order from 120, will become the radius of circles correſponding with the conjugate of each elipſis, till it arrives at the conjugate part of the curve, 1 to 5, from whence it began.

This quantity comes out in my table nearly 344, as may be ſeen on the upper part of table, 1.

The table being intended to paſs by 24 increaſing quantities, the manner of proceeding will be as follows.

From the radius of the quadrant 120-000
Take the tranſverſe radius - - 8-136

Difference 111-864

Divide the difference by 24, twice	Difference.	Mean.	Additionals.	½ Additionals
$\frac{1}{24}$)	111.864	4.661	.194	.097

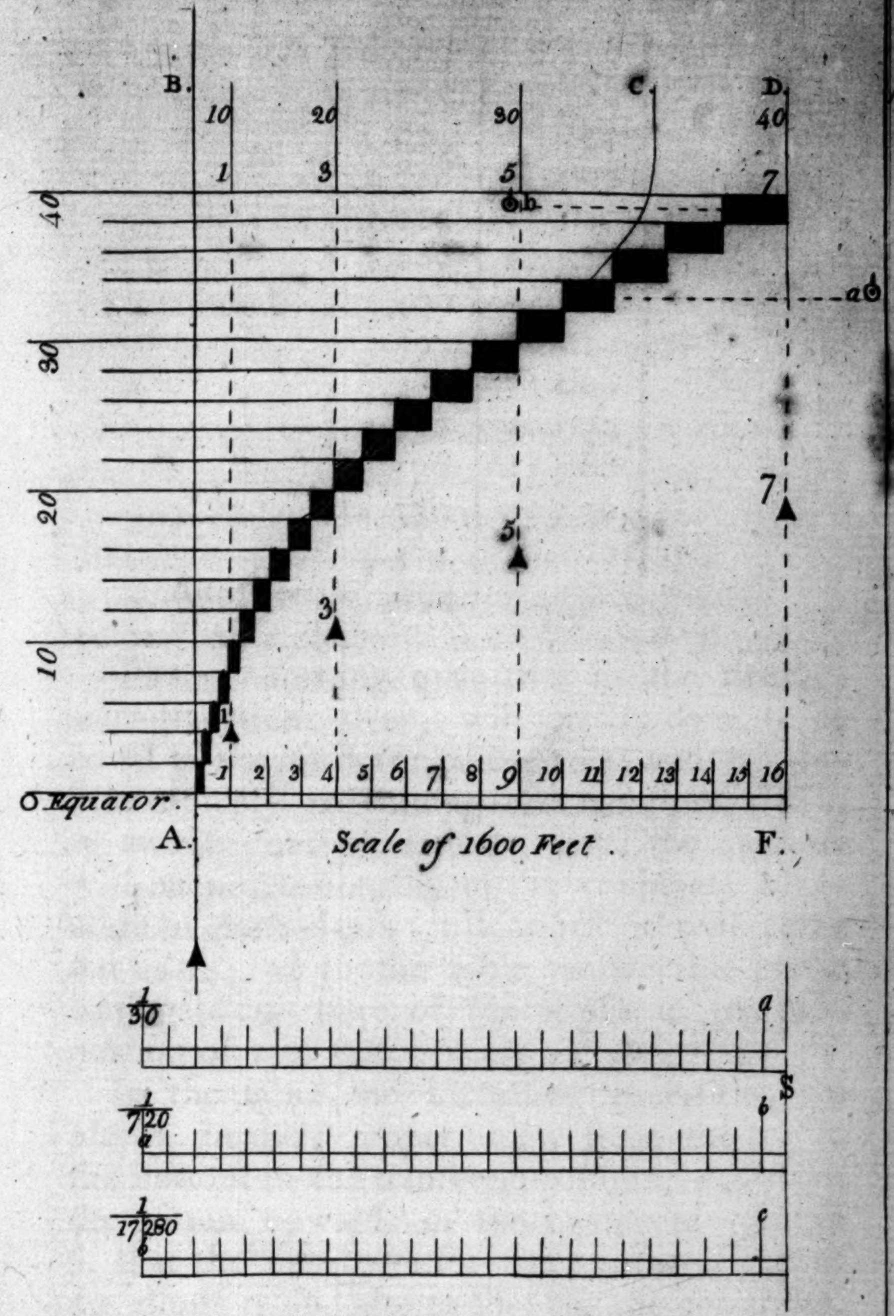
From the mean exceſs 4-661
Take half the additional -097

Remains the 12th term 4-564
From which take 11 additionals 2-134

Remains 2-430

This laſt remainder will be the beginning for a ſeries of numbers increaſing by the continual addition of, .194 proper to add to the given tranſverſe radius, and will ſtand for the firſt twelve gradations in the order following.

Plate 5


B.
C.
D.
10
20
30
40
1
3
5
7
40
30
20
10
b
a
7
5
3
1
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
Equator.
A.
Scale of 1600 Feet.
F.
1/30
a
S
1/720
a
b
1/17280
b
c

	0.000	8.136
	2.430	10 566
	2.624	13.190
	2.818	16.008
	3.012	19.020
	3.206	22.226
6	3.400	25.626
	3.594	29.220
	3.788	33.008
	3.982	36.990
	4.176	41.166
	4.370	45.536
12	4.564	50.100
Additional	.194	

Theſe increaſing quantities in the twenty-four gradations given, will exactly come to be equal to the quadrant radius 120; and by uninterruptedly continuing the ſame progreſſion to twenty-four gradations more, the increaſe will be proportionably to the conjugate curvature of each elipſis: till at laſt it will come out at 344, as in the table, anſwerable to the conjugate curvature of the elipſis in the proportion of 1 to 5.

In the table the increaſing quantities are placed, in their proper order, from the top to the bottom in the tranſverſe column, and from the bottom upwards in the conjugate column ſo that the curvature for the tranſverſe part of the elipſis is oppoſite to that for the conjugate.

Each number in the tranſverſe column is expreſſive of the proportion, the tranſverſe radius or curvature bears to the radius of the quadrant 120, and each number of the conjugate expreſſes what the conjugate, radius or curvature exceeds that of the quadrant. The oppoſite numbers compared with each other is the ratio of the curvature itſelf, in the extreme parts of each curve or ſpheroid.

This firſt table then comprizes twenty-four different proportioned curves in equal diviſions, from 1 to 5 to the quadrant; conſequently, the neareſt curve to the quadrant, in this table is that in the proportion of 29 to 30. Now ſuppoſing the firſt table did not diſcover the exact proportion of curvature in the two extremes of the flatter ſpheroids, yet, as the numbers in the ſecond table which takes in twenty-four more ſpheroids between 29 to 30, and the quadrant, would, if their progreſſion were continued; paſs through all the other numbers found in table the firſt, it is evident that they approach ſtill nearer the truth as they are nearer the radius of the quadrant, fixed at 120: ſo that the firſt table ſerves as a baſis for the ſecond. pointing out the right order of increaſe requiſite for its formation.

The ſecond table alone is ſufficient to aſcertain any proportion relative to the earth. The firſt table obtains a ſpheroid 29 to 30, which falls ſhort of the circumſcribing circle by 133.5

miles, but the ſecond, by including twenty-four ſpheroids between the above proportion of 29 to 30, and the quadrant, obtains a ſpheroid within 6 miles of the quadrant, or the earths ſurface, ſuppoſing it a perfect ſphere. The third table, which takes in twenty-four other ſpheroids, between table the ſecond, and the quadrant, obtains a ſpheroid within a quarter of a mile.

The radius correſponding with the tranſverſe curvature will always be leſs, and the conjugate radius more, than the radius of the quadrant: were the ſpheroid to approach within the thickneſs of a leaf of gold of the ſurface, and were this proceſs by tables carried on till within that thickneſs, the two curvatures compared, would be found more than three times as variable as the diameters of the ſpheroid they are upon; ſo that if the ſphere be ever ſo little oblated this alteration takes place, though before perfect, and the convexity at the pole and the equator exactly equal.

The above mentioned facts, corraborate what was advanced in the explanation of plate 3, that gravity is connected with the diameters, and not follow this change of curvature. Look into any part of tables, the 2nd, 3d, or 4th, and compare together the two numbers in a line with each other, which point out the ratio of curvature in the extreme parts of the ſpheroid. Their ratios will be found at leaſt three times as variable as the ratio of the diameters, and are ſo far from coming nearer to the diameters, in conſequence of the nearer approach of the ſpheroid to a ſphere, that they are rather more diſtant. This may be clearly ſeen by inſpecting table the 3d, where the diameters at that point which anſwers to about a quarter of a mile from the ſurface of the earth, are to each other in the proportion of 17279 to 17280, now the ratio of curvature correſponding with theſe diameters is in the proportion of 4943 to 4944, as may be ſeen by comparing the two oppoſite numbers.

What has been ſaid is abundantly ſufficient to demonſtrate that the ratio of curvature never comes nearer to the ratio of the diameters, though the proportion of the ſpheroid be ever ſo near the ſphere. That the denominators of the ratio of the diameters, and the ratio of curvature in the two extreme parts of the ſame ſpheroid, will be always nearly in the proportion to each other, as 1 to 3, and that by fixing a certain number for the quantity of the radius of the circumſcribing circle of the elipſis, and continually adding the increaſing quantities, it can be computed exactly what the radius of the tranſverſe falls ſhort, and that of the conjugate exceeds the radius of the great circle.

In conſtructing table the ſecond, do as follows.

Find the difference between 120, the radius of the ſphere, and 113.108.

Radius of ſphere	120.
The tabular number	113.108
Difference	6.892

Divide this difference by 24, and there will be produced a mean = .287166. Then divide the additional number of the firſt table which is, .194 by 24 twice, and obſerve half the laſt quotient, as follows.

	Increment.	Mean.	Additional.	half Additional.
24)	.194	.008083	.000336	.000168

From the mean exceſs - -	.287166
Take half the additional number	.000168
Remains the 12th term - -	.286998
From which take 11 additionals	.003696
Remains	.283302

This remainder will be the beginning number for a ſeries of increments by the conſtant addition of .000336 to it, which will produce quantities, continually increaſing, proper to add to 113.108 in the tranſverſe column; the neareſt number to 120, in table the firſt, then proceed as in that table.

The third table is to be proceeded with in all reſpects, the ſame as deſcribed for the preceeding.

The fourth table is conſtructed between the 9th, and 10th, numbers from the top of table 2nd; of courſe the ſame additional .000336 is uſed as in table the third: only it muſt be obſerved, that as there is more difference between the two conjugate than there is between the two tranſverſe numbers, there will be two means given to work by.

I have already obſerved, that the firſt table obtains a ſpheroid in the proportion of 29, to 30, which is within 134 miles of the quadrant, this is repreſented in three ſcales of proportion in plate 5th.

The perpendicular line S, is ſuppoſed to be the extent of radius of the perfect ſphere to the left hand of it: and the point on the upper ſcale at $\frac{1}{30}$, the conjugate part of the ſpheroid obtained by the firſt table: this will be diſtant from S 134 miles nearly.—Then as the ſecond table obtains twenty-four other ſpheroids between $\frac{1}{30}$, and S, the laſt ſpheroid obtained by the ſecond table will be at *a*, neareſt to S, or within 6 miles of the ſurface of the quadrant; and the proportion of the ſpheroid as 719, to 720.

Again. if we ſuppoſe twenty-four ſubdiviſions or ſpheroids between *a*, and S, on the upper ſcale, as given in table the 3d, the diſtance from *a*, to the line S, on the upper ſcale, muſt be enlarged to be from S to *a*, by $\frac{1}{720}$ on the middle ſcale, then will the point *b*, on the middle ſcale be the ſpheroid obtained by table the 3d, it will attain the circumſcribing ſphere within a quarter of a mile, and its proportion will be as 17279, to 17280.

In the ſame manner if the diſtance from *b*, to the line S, in the middle ſcale, were enlarged and *b*, removed to $\frac{1}{17280}$, then would *c*, near the penpendicular line, S, be the point of a conjugate part of a ſpheroid obtained by a 4th table, and would be within 53 feet of the ſurface of the quadrant. A 5th table by the ſame proceſs would produce a ſpheroid to come within 2 feet; a 6th within 1 $\frac{1}{4}$ inch; and a ſeventh within $\frac{1}{16}$ of an inch. This method is a full confirmation that 13, to 14, is the true ratio of curvature in the two extremes of the ſpheroid of the earth.

The application of theſe tables is thus:

Suppoſe and oblate ſpheroid be given in axis, 4, and in the tranſverſe 5, what is the proportion of curvature or radii, in the two extreme parts of it?

Examine the firſt table againſt $\frac{4}{5}$ in the column of the diameters ſtands, 81.558 for the tranſverſe, that is, it is $\frac{81558}{120000}$ of the radius of the great circle. Againſt $\frac{4}{5}$, likewiſe ſtands in the right hand colum, 165.426 for the conjugate, that is to ſay, it exceeds 120, the radius of the great circle, 45.426 and the two extreme numbers compared to each other are nearly as 1 to 2: which is placed againſt the other numbers in the right hand column of the table. So at $\frac{29}{30}$ diameters ſtands, $\frac{8}{9}$ for the proportion of curvature in the two extremes of ſuch a ſpheroid.—In table the ſecond, diameters $\frac{719}{720}$, and the curvature or radii, in the two extreme parts $\frac{205}{206}$ in table 3d, diameters $\frac{1439}{1440}$, curvature $\frac{411}{412}$.

Likewiſe in the 4th table, which touches the earths ſpheroid againſt $\frac{45}{46}$ diameters ſtands $\frac{11}{12}$ the true proportion of radii or curvature in the two extreme parts at the equator, and the poles. The number 120, in the tables being fixed for the ſemi diameter of the earth at the equator, by finding what that number exceeds 115.4791 and computing it, the difference which is 4.5209 anſwers to about 150 miles as inſerted in plate 4, fig. 2, where *a*, is the centre of the earth, and, *b*, the point of the radius which would neareſt correſpond with the curvature at the equator. Then by obſerving what 124.6033 exceeds 120, which is 4.6033 and anſwers to 153 miles, and adding this difference to the falling in of the pole which is 87 miles, the amount is 240 miles, which is the diſtance below the earths centre, at which the point of the radius would be, which would neareſt fit the curvature at the pole, and falls at *i*, fig. 2, plate 4. Therefore the radius anſwerable to the curvature of the earth paſſes from *b*, on the equator, by *c*, to *i*, on the axis.

The next tables to be explained are the four containing the proportional lengths of eliptic curves, to the quadrant which circumſcribes them.

The proportion which the diameter of a circle bears to the circumference, as deduced from Van Culen, will give the radius as 45.8365 to 72, the quadrant: by ſubſtracting the radius from the quadrant,—the difference will be found to be 26.1635, then by proceeding with this difference as before, through twenty-five gradations, it will ſtand as follow:

	Difference.	Mean.	Additional.
25)	26.1635	1.04654	.04186

From the mean exceſs	1.04654
Take 12 additionals	0.50234
Remains	.54420

This remainder will be a beginning number by the conſtant addition of .04186 for a ſeries of 25 unequal quantities, by the continual addition of which to 45.8365 the radius, the lengths of 24 elipſis's will be given at an equal diſtance from each other, between the radius, and the quadrant. Then ſubſtract 70.4511 the neareſt number to 72, in table the firſt, from 72, and divide the difference by 25, which will produce a mean. Then by divideing the former additional in table the firſt, by 25, twice, an additional number will be found for the ſecond table, which muſt be proceeded with as deſcribed in table the firſt.

The third table will be formed by taking the difference between 72, and the neareſt number to it in the ſecond table, and working as before. Table the fourth is generated in the ſame manner between two numbers, from the ſecond table which touch the proportion of the curve, as 45 to 46.

Theſe tables will give the proportion of any elipſis to the quadrant, when the diameters of the ſpheroid are firſt given.

Let a ſpheroid be given in the proportion of 3 in the axis, to 5 in the tranſverſe, or equatorial diameter; and ſuppoſe its ſurface an elipſis.

In table the firſt againſt $\frac{3}{5}$ in the column of diameters, ſtands 58.39 which is its proportional length to 72, the quadrant.

At $\frac{24}{25}$ in the ſame column ſtands, 70.45 to 72, the lenght of the quadrant. So in table the ſecond, againſt the diameters $\frac{96}{97}$, ſtands 71.6244 for the proportional length of the elipſis to the quadrant. And in table the third againſt $\frac{1249}{1250}$ ſtands 71.9673 for the length of the elipſis to 72 the quadrant.

Table the fourth is generated from between the, twelfth and thirteenth numbers from the top of the ſecond table; and the proportion of the earths ſpheroid is nearly in the middle of it, as found oppoſite to $\frac{45}{46}$ where there ſtands 71.1572 Were the earths curve an elipſis, this would be the length of it in proportion to the quadrant or circumſcribing circle at the equator; and one degree of ſuch a quadrant would nearly correſpond with the 85th degree of the meridian: but it is found in reality to be 20 miles more prominent than the elipſis in this latitude, which cauſes its curve to be longer in proportion to the number 72, it being very near as 71.3038 to 72, which gives a diameter at the equator of 8010 miles, and a degree of the great circle at the equator, very near 69.9 miles.

The total of the numbers in the table of meridional degrees, therefore being divided by 71.3038 and the quotient multiplied by 72, the product will be a quadrant of the equatorial circumference: which divided by 90, will give the longitudinal degree on the equator; and anſwers nearly to the 80th degree in the table of meridional degrees.

The

The next tables of which I ſhall give the explanation are thoſe which are called Curve tables. There are ſeven different ſpecimens given, and five of them are adapted to the curves, delineated in plate 4, viz. in the proportions of 1 to 5, 2 to 5, 3 to 5, 4 to 5 and 9 to 10. One is given as 1 to 2, and another as 45, to 46, the exact proportion of the earths ſpheroid.

The manner of conſtructing theſe curve tables is by making uſe of a beginning number in ſuch proportion, that it ſhall be neither more or leſs, than that when the order of number up to 90 is added to it, the laſt number in the table will be in proportion to the firſt, as the two diameters of the ſpheroid they are to repreſent; which will give a juſt diſtribution of the degrees which reſpect gravity on its ſurface.

It is obvious that in forming ſuch curve tables, diſtinct beginning numbers for theſe ſeveral proportions will be requiſite, as follows.

Spheroids.				
45 to 46	take	45	times 4095	184275.
19 to 20		19		77805
9 to 10		9		36855
4 to 5		4		16380
3 to 5				6142
1 to 2		1		4095
2 to 5				2730
1 to 5				1024

The above are the loweſt terms, or feweſt figures, by which theſe proportioned tables can be made.

The total of each table ſignifies the length of the curve line it repreſents.

The proportion of each curve to the quadrant, has been already laid down in the other tables, and alſo where the degrees of the great circle fall, on each curve.

Curvature in the extremes of a ſpheroid being always ſo widely different from the ratio of the diameters, and gravity being ſtrictly connected with the diameters, this kind of tables will ſhew at once what the degree reſpecting gravity at the tranſverſe exceeds, and that at the conjugate falls ſhort of the degree reſpecting curvature at either extreme.

By fixing 120, for a radius in the tables before deſcribed for gaining the extremes of curvature in the ſpheroid, I applied it to the earth, and laid it down in fig. 2, plate 4. Gravity alſo may be diſtinguiſhed with equal truth in theſe curve tables. Take an example from the Earths Curve.

I have already proved that the degree of the great circle in the earths curve, is nearly equal to the 80th degree of the meridian; now, all radii being in juſt proportion to the degrees given, it will follow. As this degree is to the pole degree, ſo is the radius of the great circle, to the radius of the pole degree.

And as this degree is to the meridional degree at the equator, ſo is its radius, to the radius of that meridional degree. Theſe radii computed juſtly, will fall as deſcribed, plate 4, fig. 2, the centre near *a*, being the centre of the earth: the point *d*, the radius which reſpects gravity at the equator: and the point, *g*, the radius reſpecting gravity at the pole: by which it is manifeſt, that the radius correſponding with the curvature at the equator, is not leſs than 150 miles above the earths centre, while that reſpecting gravity is only 64 miles above.—Alſo the radius anſwerable to the curvature at the pole is at leaſt 240 miles below the earths centre, while that which reſpects gravity is only 108 miles below, and exceeds the radius of the degree of the great circle only 21 miles.

The radius then anſwerable to a degree of the meridian reſpecting gravity, paſſes from, *d*, on the equator, by *b*, to *g*, on the axis; and not by that curve line nearer the centre, by *a*, from *d*, to *g*: which may be proved by computing the length of radius to the 45th degree upon the curve; and by letting it fall perpendicular to the tangent of that degree. Its extent and direction will be to the point *b*.

It ought to be obſerved that as there are 91 numbers in theſe tables, the firſt and laſt, expreſs the ſmalleſt and largeſt arc, that can be found on the ſpheroid. So in the table of meridional degrees or the curve of the earth, the firſt number affixed is no part of the curve; but it expreſſes the quantity of the degree of a ſphere that would perfectly correſpond with the curvature found at the equator, and alſo ſuch curvature as reſpects gravity.—The quantity of the other 90 numbers, ſeperate from this firſt number, is the true length of the curve.

The ſmalleſt and largeſt arcs, are always to be conſidered as equal to the two diameters, it being really ſo as reſpects gravity.

Next follows the table of longitudinal degrees, with their quantities on every degree of latitude from the equator, in Engliſh ſtatute miles, and decimal parts.

The three tables which come next are connected with each other, and refer to a method of finding the ſpheroidiſm of the earth by the obſervation of the parallax of the moon at different points in the middle latitudes, at a great extent aſunder. But, as I ſhall ſpeak of that further on, I will then refer to and explain them.

The table of the deviation of the perpendicular line from the tangent, at the ſurface at every 10th degree of latitude, from the centre of the earth, is plain by inſpection only.

Laſt of all is a table of the real quantity of the parallax of the moon as it reſpects the ſphere or ſpheroid. The difference as found on every tenth degree of latitude is laid down in the right hand column, and it is to be underſtood that the parallax is 30 ſeconds leſs on the 50th degree of latitude, than on the 50th degree of longitude from the ſame meridian upon the equator, and ſo by any other 10th degree: It muſt be remembered that this and the three above mentioned tables are conſtructed, taking the moons horizontal parallax at 60 minutes.

The principal evidences of what I at firſt propoſed, to prove being involved in the explanation of the tables, and plates in the foregoing pages, I ſhall now offer a few general obſervations which may ſerve either to inforce or illuſtrate what I have there advanced.

One of the points of the greateſt conſequence to be eſtabliſhed, is, that the degrees on the ſurface of a ſpheroid reſpect the quantity of matter contained in it, and not the curvature: this has been enlarged upon in explaining plate 3, and elſewhere.—Some years ago I was doubtful of this fact: I found that the two extremes of any ſpheroid were widely different from the proportion of its diameters, I enquired the opinions of ſeveral gentlemen of great reputation for mathematical knowledge, reſpecting this circumſtance; and found them very different: this excited me to the inveſtigation of it myſelf, and the reſult of my endeavours is comprized in my tables of the diameters, and curvature of any proportioned ſpheroid, by which I prove that the two extreme degrees, reſpecting gravity are always epual to the two diameters, but the two extreme degrees of curvvature never ſo; ſo that the largeſt and ſmalleſt arc reſpecting gravity are widely different from thoſe which reſpect curvature, my tables clearly point out the diſtinction between them.

I alſo find that if there were a million different proportioned ſpheroids between 14 to 15, and 4 to 5, there would be but one point among them in which the cubes of their diameters would be the true ratio of curvature in their extremes.

All ſpheroids more variable in their diameters than as 4 to 5, are leſs variable in their extremes than the cubes of their diameters; and all which are leſs variable than as 14 to 15, are more variable in their extremes, reſpecting curvature, than the cubes of their diameters: this will be demonſtrated more clearly by the following table, in which is inſerted a variety of proportions in a line with the cubes of their diameters.

DIAMETERS, AND CURVATURE.

Diameters.	Curvature as the Cubes of the Diameters.	From the tables of Curvature.
1	1	1
5	125	42
2	1	1
5	16	11
3	1	1
5	5	4
4	1	1.1
5	2	2.1
9	2.7	2
10	3.7	3
19	6	5
20	7	6
29	9.3	8
30	10.3	9
45	14.6	13
46	15.6	14
59	19.3	16
60	20.3	17
229	76	66
230	77	67
719	239	205
720	240	206
17279	5759	4945
17280	5760	4946
1000000	333333	285714
1000001	333334	285715

I have proved that the two extreme meridional degrees are as 46 to 45. Now, if this were the ratio of curvature, and not the ratio of the diameters, the ratio of the diameters anſwerable to this curvature, would be by the cubes $\frac{138}{139}$ which is only fifty-eight miles ſhorteſt in the axis, but as it appears from what I have ſaid that the ratio of gravity is connected with the two extreme degrees of the meridian, therefore that $\frac{45}{46}$ is the proportion of the diameters, and the ratio of curvature correſponding with this nearly $\frac{14}{15}$; conſequently as $\frac{45}{46}$ is three times as variable as $\frac{138}{139}$, the axis, is three times 58 = 174 miles ſhorter than the equatorial diameter.

Sir Iſaac Newton gives his firſt and laſt meridional degrees as 76 to 77, and his diameters as 229 to 230, and a late writer has given the firſt and laſt degrees as 59 to 60, and the diameters as 178 to 179, from which it is plain that they have taken it for granted, that meridional degrees originate their quantities from the ratio of curvature, becauſe $\frac{229}{230}$ are the diameters anſwerable to $\frac{76}{77}$ curvature.

Here then principally ariſes this immenſe miſtake, by taking the ratio of curvature for gravity.—Sir Iſaac Newton's proportion of $\frac{77}{76}$ ought to have been the diameters, and the ratio of curvature three times as variable, which would have brought the earth 102 miles ſhorteſt in its axis inſtead of 34 miles, and the other

propor-

proportion of $\frac{59}{60}$ would make the earth 133 miles ſhorteſt in its axis inſtead of 44.

That gravity is not connected with curvature might be proved, by a method different from what I have laid down in the foregoing part of this pamplet will explain it as briefly as poſſible.

Let ſeveral different proportioned ſpheroids be given, as in the proportions of 1 to 5, 2 to 5, 3 to 5, 4 to 5, and 9 to 10. A quadrant of a ſection in each of theſe proportions may be found in plate 4, fig. 1.

Then refer to the firſt table of the diameters and curvature of ſpheroids, and obſerve what the two radii in the extreme parts of each of the above proportions, exceeds two radii of the circle which circumſcribes each ſpheroid, ſuppoſing them all equal in the tranſverſe diameter, for example, oppoſite to $\frac{1}{5}$ diameters ſtand 8.136, and 343.608, which added together make nearly 352: but twice 120 the radius of the quadrant is but 240, this makes the exceſs of the extremes 112, which is almoſt half as much again, though the ſpheroid given contains but about $\frac{1}{5}$ of the quantity of matter contained in an undeminiſhed ſphere of the ſame diameter as its tranſverſe part.—This ſingle circumſtance conſidered impartially is ſufficient to evince the impoſſibility of gravity being connected with curvature.

But if the degrees on the ſurface be diſtributed in the order reſpecting gravity, according to the curve table which repreſents a curve in the proportion of that we are ſpeaking of, viz. as 1 to 5, the two radii in the extremes, when compared with two radii of the circumſcribing circle, will be equal but to 1 radius, and $\frac{2}{3}$ of that circle.

This as gravity reſpects matter is very conſiſtent; for the two extreme radii diminiſh with the quantity of matter: but if conſidered as curvature they greatly increaſe even while the quantity of matter diminiſhes.

If a perfect ſphere were formed from the quantity of matter contained in the above mentioned flat ſpheroid, two of its radii would be found to be equal to 1 radius and $\frac{1}{6}$ of the circumſcribing circle. Proceed in the ſame manner with the other ſpheroids in the proportion of 2 to 5, 3 to 5, &c, and the reſult will be nearly as in the following table.

Proportion of ſpheroid.			Two radii when collected into a perfect ſphere.	Two radii reſpecting gravity.	Two radii reſpecting Curvature.
1		5	$1 - \frac{1}{6}$	$1 - \frac{2}{3}$	$2 - \frac{93}{100}$
2		5	$1 - \frac{2}{3}$	$1 - \frac{3}{4}$	$2 - \frac{52}{100}$
3	to	5	$1 - \frac{3}{5}$	$1 - \frac{4}{5}$	$2 - \frac{23}{100}$
4		5	$1 - \frac{4}{5}$	$1 - \frac{7}{8}$	$2 - \frac{5}{100}$
9		10	$1 - \frac{7}{8}$	$1 - \frac{16}{17}$	$2 - \frac{14}{1000}$

This table exhibits the proportion which two radii of the extremes of each ſpheroid bear to two radii of the circumſcribing circle; from which it is obvious, that the enormous increaſe of the two radii reſpecting curvature, while the quantity of matter diminiſhes, is an extravagant abſurdity; and cannot poſſibly take place as gravity reſpects matter.

It is certain from what has been ſaid that two degrees in the extreme parts of a ſpheroid reſpecting gravity, never exceed two degrees or radii of the circumſcribing circle or undiminiſhed ſphere. The degree of gravity in all ſpheres increaſes but by a two fold, whilſt the quantity of matter increaſes by an eight-fold proportion.

In all ſpheroids, the point to which the plummet is attracted both at the conjugate, and tranſverſe, lies between the point correſponding with the curvature and the common centre of gravity, and it is always nearer at the conjugate, to the common centre of gravity than the centre of curvature.

But at the tranſverſe of flat ſpheroids, the point to which the plummet, is attracted is nearer the centre of curvature than the common centre of gravity, or the centre of the ſpheroid. All ſpheroids approaching nearer to the ſphere than 9 to 10, are excepted from this; in ſuch, the plummet at the tranſverſe as well as the conjugate, tends to a point nearer the centre of the ſpheroid than the centre of curvature.

Two portions of ſpace in the two extremes of the ſurface of any proportioned ſpheroid perfectly conſiſtent with gravity may be obtained by this method.

In fig. 1, plate 4, from the axis at 90 ſet off one degree, and draw a line from thence to the centre *o*. next move the ſpace where any of the curves in the plate touch the degree before drawn to the tranſverſe end of each curve at *o*. then bring down the degree of the great circle at 90 and place it in its room, and it is done, both theſe ſpaces will form equal areas to the centre *o*, in any proportion as can be named.

The following method will obtain the ſurface of the ſpheroid much nearer to the real curve of the earth than an elipſis.

Set in the oblatiſm or falling in of the pole on the axis at 90, of fig. 1, plate 1, and divide it into four parts. Take five ſuch parts, and ſet them off from the centre *g* to *e*: next take ten ſuch parts, and ſet them off from *g* to *f* on the axis, and draw a right line through the points *f*, and *e*, towards the 60th degree on the ſurface. From the centre *e* draw the arc from *o*, to near the 60th degree, and the remainder of the 90 from the centre *f* on the axis, and both theſe centres will fall between the centre of the ſpheroid, and the centre which will neareſt correſpond with the curvature.

The method already laid down in this tract of obtaining the true ſpheroidiſm of the globe will perform it to a certainty, but ſeveral other ways might be employed which though they cannot be brought to ſo great a degree of exactneſs as that, may ſerve as a confirmation of its reſult. The following proceſs may not be unreaſonable. Let the diameter magnitude and time of the diurnal rotation of the planet, Jupiter be given, and ſuppoſe the earth to be 8000 miles in diameter, then proceed as follows.

If ten hours, which is nearly the time of one revolution of Jupiter on its axis, acquire $\frac{12}{13}$, which is his ſpheroidiſm; what ſpherodiſm will 24 hours acquire: it will be found to be nearly $\frac{30}{31}$, which will give 289 miles ſhorteſt in the axis: but this proportion will not hold ſtrictly true; for the Earth would be ſo far from acquiring $\frac{12}{13}$ ſpheroidiſm, by revolving on its axis in 10 hours, that it would be requiſite for it to be but four hours revolving to acquire that ſpheroidiſm: which may be demonſtrated in the manner following.

Divide the diameter of Jupiter continually by 5, and the time of its revolution by 2, till much ſmaller magnitudes are attained; by which the ratios deducible will ſtand as in the table annexed.

Spheroidiſm by Centrifugal force.

	Miles Diameter.	Time of one Revolution. Minutes.	 Seconds.	Ratios deducible. Miles Diamet.	 H. ~~Minutes.~~	 M. ~~Seconds.~~
	89000.	600.	0	8000	4	
	17800.	300.	0	1000	1	40
Earth.	3560.	150.	0	100	0	35
	712.	75.	0	10	0	11
	142.	37.	0	1		4
	28.	18.	0			
	5.7	9.	3			
	1.1	4.	6			
	Feet Diameter.			Feet Diameter.	M.	S.
	1160.	2.	3	1000	2	
	232.	1	17	100	0	50.
	46.4	0	38	10	0	20
	9.3	0	19	2	0	10
	Hours.	Revolution.		Spheroidiſm,		
	3			9	to	10
	6			19		20
	12	1		29		30
	24			39		40

Thus to acquire $\frac{12}{13}$ the ſpheroidiſm of Jupiter, it would be neceſſary that a globe 8000 miles in diameter ſhould be only four hours revolving on its axis; one of 1000 miles diameter one hour and 40 minutes: one of 100 miles 35 minutes: one of 10 miles 11 minutes. and one of 1 mile diameter 4 minutes, or fifteen revolutions in an hour: and ſo on.

Theſe deductions are ſufficiently exact to prove that a globe of the diameter of the earth, would be between 3 and 5 hours revolving on its axis, to acquire $\frac{12}{13}$ ſpheroidiſm: and if it be fixed at ſomething leſs than four hours, there is one thing more gained to work from, beſides the firſt three things given.—Then by reaſonably ſuppoſing the Earth to be 3 hours revolving once to acquire $\frac{9}{10}$ ſpheroidiſm, I deduce the laſt proceſs at the bottom of the table, that is 3, 6, 12, 24, falling in geometrical proportion; for 3 is involved into 24 eight-times, 6 four-times, and 12 twice, and the variation of the ſpheroidiſm paſſing but in three equal diviſions againſt it, brings the matter near the truth: for the earths ſpheroidiſm is found to be nearly $\frac{39}{40}$, which is about 200 miles ſhorteſt in its axis.

Thoſe who have given the earths ſpheroidiſm ſo little in proportion to that of Jupiter, have not reaſoned well: for if his equatorial ſurface is ten times more diſtant from his axis than that of the earth, his poles axis is in proportion to counteract and balance that enormous power: and though it muſt be allowed that the greater the body is the more ſpheroidiſm it will acquire in the ſame time of revolution,

tion, yet it is plain the difference is by no means ſo great as many have imagined.

The ſpheroidiſm of the earth may be obtained, tolerably near, by nice obſervations of the parrallax of the Moon, at diſtant parts of the Earth, and at the ſame time.

On a large diagram draw a quadrant and ſeveral curves in the ſame proportions as laid down in fig. 1, plate 4: and from the reſpective table of each curve, transfer the numbers in juſt proportion to each curve line, then will every degree fall at its proper point as it reſpects gravity. Next divide the radius from *o* to 90, on the axis into 60 equal parts, then raiſe a perpendicular from the ſurface by *o*, parellel to the axis, and ſet up 59 of the 60 equal parts on it; the dificient part being allowed for the diminution of the Moons parallax in the diſtance diminiſhed by the ſemi-diameter of the Earth: let theſe 59 parts be divided into 60 equal parts, as before; and by drawing lines from one to the other, a true ſcale of every minute of the Moons parallax will be obtained, as it will fall upon each curve, or different proportioned ſpheroid.

By this means it will be diſcovered, not only how much more the angle of the Moons parallax is diminiſhed by the oblatiſm at the pole (as from *a* to *b*, fig. 4, plate 3,) than it is at the equator, but alſo in any latitude whatever; as at the ordinates 80, 60, 40, and 20 in the ſame figure. It is found to be at Greenwich about 30 ſeconds of parallax, which anſwers to 50 miles upon the Earths ſurface, leſs than on the equator.

Were it poſſible that two obſervers could take an obſervation of the Moon at the pole, and ninety degrees from a fixed meridian on the equator at the inſtant the Moon is on that meridian, and in the equator, the difference in the whole horizontal parallax of the two obſervations, would give the ſpheroidiſm, but this would be impracticable.

However, the ſpheroidiſm might be obtained tolerably near, if the above mentioned obſervation were performed in any high latitude, between 60 and 70 degrees, and the number of degrees and minutes from the ſame meridian on the equator, and when the Moon is over that meridian, and on the equator, from the difference between the quantity of parallax of the two obſervations.

But as ſuch a method would be attended with much trouble and expence, I will mention another, which would obtain it nearly as accurate, with repeated and exact obſervations.

I will fix upon the ſpace between the 40th and 60th degrees of latitude upon the ſame meridian. Note the proportional length of each curve, on the before mentioned diagram, to the quadrant, and particularly obſerve the quantity of the Moons parallax that is found between the 40th and 60th degrees on each curve; as well as on the quadrant or perfect ſphere: and it will be found to be nearly as placed under diff. in the table of the difference of the Moons parallax between the 40th and 60th degrees of latitude in various proportioned ſpheroids.

Theſe points being gained, will be ſufficient to graduate a firſt table from, as a baſis for a ſecond, that will include the proportion of the curve of the Earth.

At the top of table the firſt is 2.6352, and at bottom for the perfect ſphere is 13.1760 this firſt table nearly attains the quantity of the difference of parallax between the 40th and 60th degrees on each curve or ſpheroid; and alſo diſcovers the difference in a ſpheroid as 29 to 30, which is the neareſt number to a ſphere and is found to be 12.738.

Proceed to graduate between the laſt number found and the ſphere, as in table 2, and the proportion of the Earths curve will be paſſed, which will fall near the 9th number from the top, and his found to be neareſt 12.8840. The application is as follows.

When the Moon is ſo near to the Earth, that its whole horizontal parallax is exactly

60 minutes, the difference of parallax between the 40th and 60th degrees of latitude, ſuppoſing the Earth a perfect ſphere is exactly 13.1760 minutes, but becauſe of the oblatiſm, this will be ſenſibly diminiſhed ; and the curve as 45 to 46 gives it but 12.884 minutes, the difference is .292 parts of a minute, which multiplied by 60, produces 17.52 ſeconds, which is the difference between the ſpheroid and the perfect ſphere, between the two points given: this difference will give the ſpheroidiſm.

I have already proved that the equatorial diameter of the Earth is 8010 miles, and it can be eaſily known when its ſemi-diameter will agree to 60 minutes horizontal parallax.

The following is the difference by the firſt table between the degrees given.

In the ſpheroid	**29 to**	**30**	**the difference is**	**.4392**	**parts of a min.**
In that as	**9 to**	**10**	**the difference is**	**1.3176**	**minute**
In that as	**4 to**	**5**	**the difference is**	**2.6352**	**minutes.**
In that as	**3 to**	**5**	**the difference is**	**5.2704**	**minutes.**
In that as	**2 to**	**5**	**the difference is**	**7.9056**	**minutes, &c.**

I will here conclude my remarks on this important ſubject, by inſerting the ſeveral diameters of the Earth at the places under mentioned, and the difference between each of them and the equatorial diameter.

	Miles.		Diff. miles.
Diameter at the Equator	8010	—	00
20th degree of latitude	8001	—	09
41ſt deg. Conſtantinople	7974	—	36
51ſt degree Greenwich	7953	—	57
60th deg. Peterſburgh	7933	—	77
67th degree Kittis	7924	—	86
90th the Poles axis	7836	—	174

UNIVESAL STANDARD, FOR WEIGHTS AND MEASURES.

TO obtain an univerſal ſtandard for weights and meaſures, having for ſome years paſt been an object of attention, and no method perfectly ſatisfactory having yet been made public, I beg leave to offer one here; which I conceive to be effectual for all the purpoſes intended. I will proceed to lay it down as briefly as poſſible.

I have demonſtrated from the generated curve fig. 2, plate 1, that the 52nd degree on which London ſtands, meaſures 365472 feet, Engliſh ſtatute meaſure, which is within 200 feet of 69. ¼ miles. Now if this degree were meaſured at any future period, and its whole quantity divided into 365472 equal parts, the length of the preſent Engliſh foot will be neceſſarily obtained; for it is certain, that the degree might be meaſured at any time within one or two ſeconds, and one ſecond is not quite 102 feet; but ſuppoſing the inacuracy of meaſure to amount to two ſeconds, and even to three, it would not amount in ſo ſmall a ſubdiviſion as the Engliſh foot, to the thickneſs of a ſheet of paper: therefore an univerſal compariſon of foreign meaſures with our foot being made and recorded, all thoſe meaſures might be juſtly obtained, ten thouſand years hence.

For weights, let this foot be cubed, and filled with Sea or Ocean water, noting particularly that part of the Ocean it was taken from; which ſhould be ſome leagues from land: and let this be a weight to compare all other known weights by,—and the whole being exactly recorded, all theſe weights may be found at any time whatever.

Or let the 100.000th part of the 52nd meridional degree after having been accurately found, be called a Geographical yard; and the 10th of each yard, or the millionth part of the degree, be called a Geographical foot: and let theſe meaſures be an univerſal ſtandard of compariſon for all known meaſures, and this method will be as effectual as the above.

And for weight: let the Geographical yard acquired as above, be cubed and filled with Ocean water as before mentioned, and called a Geographical ton weight: and let the cubic contents of the Geographical foot be called a Geographical pound, which will be exactly the $\frac{1}{1000}$th part of the ton, and if theſe weights were to be compared with all known weights, and due record made of the tranſaction, the deſired end would be obtained.

All ſmaller weights might be decimally deduced from the larger, and ſmall meaſures the ſame. Suppoſe in ſome ſuch order as the following.

Table of Geographical meaſure.

10 Parts - -	1 Inch
10 Inches -	1 Foot
10 Feet - -	1 Yard
10 Yards -	1 Chain
10 Chains -	1 Furlong
10 Furlongs	1 Mile
100 Miles -	1 Degree

Table of Geographical weight.

10 Parts - - - 1 Grain
10 Grains - - 1 Dram
10 Drams - - 1 Ounce
10 Ounces - - 1 Pound
10 Pounds - 1 Stone
10 Stones - - 1 Hundred
10 Hundreds - 1 Ton

All kinds of calculation would be much eaſier in ſuch diviſions as theſe than by the common mode.

What I call the Geographical yard is nearly equal to 3 foot 7 inches $\frac{7}{8}$ preſent meaſure, and the foot, to 4 inches $\frac{37}{96}$.

The ton weight to 24C, and the pound to 2lb. 11oz. of Avoirdupoiſe.

I might add much more, but what I have ſaid is ſufficient to point out the reaſonableneſs and utility of the method, both for eſtabliſhing true ſtandards, and for ſhortening the preſent mode of calculation.

FINIS.

BRITISH MUSEUM

ERRATA.

Page vii, line 3, *for* Parallelax, *read* Parallax
Page viii, line 16, *for* tothe, *read* to the
Page 4, *at the bottom leave out* too
Page 34, l. 15, for *Explanatiou*, r. *Explanation.*
Page 47, line 25, *for* and *read* an
Page 49, line 19, *for* follow: *read* follows:
Page 60, l. 26, *for* undeminiſhed, *r.* undiminiſhed.

TABLE of the Quantity of each Meridional Degree in Feet and Parts, and alſo the reſpective Increment which every Degree increaſes by, from the Equator to the Pole.

Deg.	Ft.	P.	Feet	Parts	Deg	Ft.	P.	Feet	Parts.
0	0	000	362794	000	45	87	480	364806	040
	1	944	362795	944		89	424	364895	464
	3	888	362799	832		91	368	364986	832
	5	832	362805	664		93	312	365080	144
	7	776	362813	440		95	256	365175	400
5	9	720	362823	160	50	97	200	365272	600
	11	664	362035	824		99	144	365371	744
	13	608	362848	432		101	088	365472	832
	15	552	362863	984		103	032	365575	864
	17	496	362881	480		104	976	365680	840
10	19	440	362900	920	55	106	920	365787	760
	21	384	362922	304		108	864	365896	624
	23	328	362945	632		110	808	366007	432
	25	272	362970	904		112	752	366120	184
	27	216	362998	120		114	696	366234	880
15	29	160	363027	280	60	116	640	366351	520
	31	104	363058	384		118	584	366470	104
	33	048	363091	432		120	528	366590	632
	34	992	363126	424		122	472	366713	104
	36	936	363163	360		124	416	366837	520
20	38	880	363202	240	65	126	360	366963	880
	40	824	363243	364		128	304	367092	184
	42	768	363285	832		130	248	367222	432
	44	712	363330	544		132	192	367354	624
	46	656	363377	200		134	136	367488	760
25	48	600	363425	800	70	136	080	367624	840
	50	544	363476	344		138	024	367762	864
	52	488	363528	832		139	968	367902	832
	54	432	363583	264		141	912	368044	744
	56	376	363639	640		143	856	368188	600
30	58	320	363697	960	75	145	800	368334	400
	60	264	363758	224		147	744	368482	144
	62	208	363820	432		149	688	368631	832
	64	152	363884	584		151	632	368783	464
	66	096	363950	680		153	576	368937	040
35	68	040	364018	720	80	155	520	369092	560
	69	984	364088	704		157	464	369250	024
	71	928	364160	632		159	408	369409	432
	73	872	364234	504		161	352	369570	784
	75	816	364310	320		163	294	369734	080
40	77	760	364388	080	85	165	240	369899	320
	79	704	364467	784		167	184	370066	504
	81	648	364549	432		169	128	370235	632
	83	592	364633	024		171	072	370406	704
	85	536	364718	560		173	016	870579	720
45	87	804	364806	040	90	174	904	370754	680

Proportion of Curvature in the two Extremes, to the Diameters of Spheroids.

TABLE I.

	Tranſverſe.	Diameter	Conjugate.	Curvature.
0	8 136	1/5	343 608	1/42
	10 566		332 060	
	13 190		320 706	
	16 008		309 546	
	19 020		298 580	
	22 626		287 808	
6	25 626	2/5	277 230	1/11
	29 220		266 846	
	33 008		256 656	
	36 990		246 660	
	41 166		236 858	
	45 536		227 250	
12	50 100	3/5	217 836	1/4
	54 858		208 616	
	59 810		199 590	
	64 956		190 758	
	70 296		182 120	
	75 830		173 676	
18	81 558	4/5	165 426	1/2
	87 480		157 370	
	93 596		149 508	
	99 906	9/10	141 840	2/3
	106 410		134 366	
	113 108	29/30	127 086	8/9
24	120 000		120 000	

TABLE II.

	Trnſverſe.	Diameter	Conjugate.	Curvature.
0	113 108	29/30	127 086	8/9
	113 391		126 786	
	113 674		126 487	
	113 958		126 159	
	114 243		125 891	
	114 527		125 593	
6	114 812		125 295	
	115 098		124 998	
	115 383		124 702	
	115 669		124 405	
	115 956		124 109	
	116 242		123 813	
12	116 529	59/60	123 518	17/18
	116 817		123 223	
	117 104		122 928	
	117 392		122 634	
	117 681		122 340	
	117 969		122 046	
18	118 258	119/120	121 753	34/35
	118 248		121 460	
	118 837		121 167	
	119 127	239/240	120 875	68/69
	119 418		120 583	
	119 708		120 291	205/206
24	120 000	726/737	120 000	

Proportion of Curvature in the two Extremes, to the Diameters of Spheroids.

TABLE III.

	Tranſverſe.	Diameter	Conjugate.	Curvature.
0	119 7089	$\frac{7}{7}$ $\frac{9}{0}$	120 2913	$\frac{205}{206}$
	119 7210		120 2792	
	119 7332		120 2671	
	119 7453		120 2549	
	119 7574		120 2428	
	119 7695		120 2306	
6	119 7816		120 2185	
	119 7938		120 2063	
	119 8059		120 1942	
	119 8180		120 1820	
	119 8301		120 1699	
	119 8423		120 1578	
12	119 8544	$\frac{1439}{1440}$	120 1456	$\frac{411}{412}$
	119 8665		120 1335	
	119 8786		120 1213	
	119 8908		120 1092	
	119 9029		120 0970	
	119 9150		120 0849	
18	119 9272	$\frac{2879}{2880}$	120 0728	$\frac{824}{825}$
	119 9393		120 0606	
	119 9514		120 0485	
	119 9635	$\frac{5759}{5760}$	120 0364	$\frac{1645}{1646}$
	119 9757		120 0242	
	119 9878		120 0121	$\frac{4943}{4944}$
24	120 0000		120 0000	

TABLE IV.

	Tranſverſe.	Diameter	Conjugate	Curvature.
0	115 3838	$\frac{44}{45}$	124 7021	
	115 2957		124 6898	
	115 4076		124 6774	
	115 4195		124 6651	
	115 4314		124 6527	
	115 4433		124 6403	
6	115 4553		124 6380	
	115 4672		124 6156	
	115 4791	$\frac{45}{46}$	124 6033	$\frac{13}{14}$
	115 4910		124 5909	
	115 5029		124 5786	
	115 5148		124 5662	
12	115 5267		124 5539	
	115 5387		124 5415	
	115 5506		124 5292	
	115 5625		124 5168	
	115 5744	$\frac{46}{47}$	124 5045	
	115 5863		124 4921	
18	115 5985		124 4798	
	115 6102		124 4674	
	115 6221		124 4551	
	115 6340		124 4428	
	115 6459		124 4304	
	115 6579		124 4181	
24	115 6698	$\frac{47}{48}$	124 4057	

TABLES of the Proportionable Length of Ellipſis's to the Quadrant, Radius 45,8365. Quadrant 72,0000.

TABLE I.

Curves	Length of Ellipſis.	
$\frac{0}{0}$	45	8365
	46	3806
	46	9667
	47	5946
	48	2644
$\frac{1}{5}$	48	9761
	49	7296
	50	5249
	51	3622
	52	2413
$\frac{2}{5}$	53	1622
	54	1250
	55	1297
	56	1763
	57	2647
$\frac{3}{5}$	58	3949
	59	5671
	60	7810
	62	0369
	63	3346
$\frac{4}{5}$	64	6742
	66	0556
$\frac{9}{10}$	67	4789
	68	9441
	70	4511
$\frac{5}{5}$	72	0000

TABLE II.

Curves	Length of Ellipſis.	
$\frac{24}{25}$	70	4511
	70	5122
	70	5734
	70	6347
	70	6961
	70	7575
$\frac{32}{33}$	70	8190
	70	8805
	70	9422
	71	0038
	71	0656
	71	1274
$\frac{48}{49}$	71	1893
	71	2513
	71	3133
	71	3754
	71	4375
	71	4998
$\frac{96}{97}$	71	5620
	71	6244
	71	6868
	71	7493
	71	8119
	71	8745
$\frac{624}{625}$	71	9372
	72	0000

TABLES of the Proportionable Length of Ellipſis's to the Quadrant, Radius 45,8365. Quadrant 72,0000.

TABLE III.

Curves	Length of Ellipſis.
$\frac{624}{625}$	71 9372
	71 9397
	71 9422
	71 9447
	71 9472
	71 9497
	71 9523
	71 9548
	71 9573
	71 9598
	71 9623
	71 9648
$\frac{1249}{1250}$	71 9673
	71 9698
	71 9723
	71 9748
	71 9773
	71 9799
	71 9824
	71 9849
	71 9874
	71 9899
	71 9924
	71 9949
$\frac{15624}{12625}$	71 9974
	72 0000

TABLE IV.

Curves	Length of Ellipſis.
	71 1274
	71 1299
	71 1324
	71 1348
	71 1373
	71 1398
	71 1423
	71 1448
	71 1472
	71 1497
	71 1522
	71 1547
$\frac{45}{46}$	71 1572
	71 1597
	71 1622
	71 1647
	71 1672
	71 1697
	71 1722
	71 1746
	71 1771
	71 1796
	71 1821
	71 1847
	71 1872
	71 1893

CURVE TABLES.

4 to 5				9 to 10			
0	16380	45	17415	0	36855	45	37890
	16381		17461		30856		37936
	16383		17508		36858		37983
	16386		17556		36861		38031
	16390		17605		36865		38080
5	16395	50	17655	5	36870	50	38130
	16401		17706		36876		38181
	16408		17758		36883		38233
	16416		17811		36891		38286
	16425		17865		36900		38340
10	16435	55	17920	10	36910	55	38395
	16446		17976		36921		38451
	16458		18033		36933		38508
	16471		18091		36946		38566
	16485		18150		36960		38625
15	16500	60	18210	15	36975	60	38685
	16516		18271		36991		38746
	16533		18333		37008		38808
	16551		18396		37026		38871
	16570		18460		37045		38935
20	16590	65	18525	20	37065	65	39000
	16611		18591		37086		39066
	16633		18658		37138		39133
	16656		18726		37131		39201
	16680		18795		37155		39270
25	16705	70	18865	25	37180	70	39340
	16731		18936		37206		39411
	16758		19008		37233		39483
	16786		19081		37261		32556
	16815		19155		37290		39630
30	16845	75	19230	30	37320	75	39705
	16876		19306		37351		39781
	16908		12383		37383		39858
	16941		19461		37416		30936
	16975		19540		37450		40015
35	17010	80	19620	35	37485	80	40095
	17046		19701		37521		40176
	17083		19783		37558		40298
	17121		19866		37596		40341
	17160		19950		37635		40425
40	17200	85	20035	40	37675	85	40510
	17241		21121		37716		40596
	17283		20208		37758		40683
	17326		20296		37801		40771
	17370		20385		37845		40860
45	17415	90	20475	45	37890	90	40950

CURVE TABLES.

1 to 5				2 to 5			
0	1024	45	2059	0	2730	45	3765
	1025		2105		2731		3811
	1027		2152		2733		3858
	1030		2200		2736		3906
	1034		2249		2740		3955
5	1039	50	2299	5	2745	50	4005
	1045		2350		2751		4056
	1052		2402		2758		4108
	1060		2455		2766		4161
	1069		2509		2775		4215
10	1079	55	2564	10	2785	55	4270
	1090		2620		2796		4326
	1102		2677		2808		4383
	1115		2735		2821		4441
	1129		2794		2835		4500
15	1144	60	2854	15	2850	60	4560
	1160		2915		2866		4621
	1177		2977		2883		4683
	1195		3040		2901		4746
	1214		3104		2920		4810
20	1234	65	3169	20	2940	65	4875
	1255		3235		2961		4941
	1277		3302		2983		5008
	1300		3370		3006		5076
	1324		3439		3030		5145
25	1349	70	3509	25	3055	70	5215
	1375		3580		3081		5286
	1402		3652		3108		5358
	1430		3725		3136		5431
	1459		4799		3165		5505
30	1489	75	3874	30	3195	75	5580
	1520		3950		3226		5656
	1552		4027		3258		5733
	1585		4105		3291		5811
	1619		4184		3325		5890
35	1654	80	4264	35	3360	80	5970
	1690		4345		3396		6051
	1727		4427		3433		6133
	1765		4510		3471		6216
	1804		4594		3510		6300
40	1844	85	4679	40	3550	85	6385
	1885		4765		3591		6471
	1927		4852		3633		6558
	1970		4940		3676		6646
	2014		5029		3720		6735
45	2059	90	5119	45	3765	90	6825

CURVE TABLES.

1 to 2				3 to 5			
0	4095	45	5130	0	6142	45	7177
	4096		5176		6143		7223
	4098		5223		6145		7270
	4101		5271		6148		7318
	4105		5320		6152		7367
5	4110	50	5370	5	6157	50	7417
	4116		5421		6163		7468
	4123		5473		6170		7520
	4131		5526		6178		7573
	4140		5580		6187		7627
10	4150	55	5635	10	6197	55	7682
	4161		5691		6208		7738
	4173		5748		6220		7795
	4186		5806		6233		7853
	4200		5865		6247		7912
15	4215	60	5925	15	6262	60	7972
	4231		5986		6278		8033
	4248		6048		6295		8095
	4266		6111		6313		8158
	4285		6175		6332		8222
20	4305	65	6240	20	6352	65	8287
	4326		6306		6373		8353
	4348		6373		9395		8420
	4371		6441		6418		8488
	4395		6510		6442		8557
25	4420	70	6580	25	6467	70	8627
	4446		6651		6493		8698
	4473		6723		6520		8770
	4501		6796		6548		8843
	4530		6870		6577		8917
30	4560	75	6945	30	6607	75	8992
	4591		7021		6638		9068
	4623		7098		6670		9145
	4656		7176		6703		9223
	4690		7255		6737		9302
35	4725	80	7335	35	6772	80	9382
	4761		7416		6808		9463
	4798		7498		6845		9545
	4836		7581		6883		9628
	4875		7665		6922		9712
40	4915	85	7750	40	6962	85	9797
	4656		7836		7003		9883
	4998		7923		7045		9970
	5041		8011		7088		10058
	5085		8100		7132		10147
45	5130	90	8190	45	7177	90	10207

	45	to	46
0	184275	45	185310
	184276		185356
	184278		185403
	184281		185451
	184285		185500
5	184290	50	185550
	184296		185601
	184303		185653
	184311		185706
	184320		185760
10	184330	55	185815
	184341		185871
	184353		185928
	184366		185986
	184380		186045
15	184395	60	186105
	184411		186166
	184428		186228
	184446		186291
	184465		186355
20	184485	65	186420
	184506		186486
	184528		186553
	184551		186621
	184575		186690
25	184600	70	186760
	184626		186831
	184653		186903
	184681		186976
	184710		187050
30	184740	75	187125
	184771		187201
	184803		187278
	184836		187356
	184870		187435
35	184905	80	187515
	184941		187596
	184978		187678
	185016		187761
	185055		187845
40	185095	85	187930
	185136		188016
	185178		188103
	185221		188191
	185265		188280
45	185310	90	188370

Quantities of the Longitudinal Degrees on every Degree of Latitude in English Statute Miles of 5280 Feet and Parts of a Mile.

Lat.	Miles	Parts	Lat.	Miles	Parts
0	69	9000	45	49	6420
	69	8804		48	7414
	69	8412		47	8212
	69	7824		46	8814
	69	7040		45	9220
5	69	6060	50	44	9430
	69	4884		43	9444
	69	3512		42	9262
	69	1944		41	8884
	69	0180		40	8310
10	68	8220	55	39	9800
	68	6064		38	9800
	68	3712		37	9725
	68	1165		36	9650
	67	8420		35	9575
15	67	5480	60	34	9500
	67	2344		33	8445
	66	9012		32	7390
	66	5494		31	6335
	66	1780		30	5280
20	65	7870	65	29	4225
	65	3764		28	3170
	64	9462		27	2115
	64	4964		26	1060
	64	0270		25	0005
25	63	5380	70	23	8950
	63	0294		22	7255
	62	5012		21	5560
	61	9534		20	3865
	61	3860		19	2170
30	60	7990	75	18	0475
	60	1924		16	8780
	59	5662		15	7085
	58	9204		14	5390
	58	2550		13	3695
35	57	5700	80	12	0000
	56	8654		11	0000
	56	1412		9	7820
	55	3974		8	5640
	54	6340		7	3460
40	53	8510	85	6	1280
	53	0484		4	9100
	52	2262		3	6920
	51	3844		2	4740
	50	5230		1	2560
45	49	6420	90	0	0000

Differences of the Moons parallax between th 40th and 60th Degrees of Latitude, on various proportioned Spheroids.

Spheroids.	Deg.	M.	S.	Deg.	M.	S.	Diff.	
1/5	40	8	0	60	10	30	2	30
2/5	40	16	0	60	21	00	5	00
3/5	40	24	10	60	31	30	7	20
4/5	40	31	30	60	42	00	10	30
9/10	40	35	40	60	47	30	11	50
Sphere.	40	39	10	60	52	20	13	10

Difference of the Moons Parallax on various proportioned Spheroids.

Propor. of Spher.	Table I.		Propor. of Spher.	Table II.	
	M.	Parts.		M.	Parts.
1/5	2	6352	29/30	12	7368
	3	0744		12	7551
	3	5136		12	7734
	3	9528		12	7917
	4	3920		12	8100
	4	8312		12	8283
2/5	5	2704		12	8466
	5	7096		12	8649
	6	1488	45/46	12	8832
	6	5880		12	9015
	7	0272		12	9198
	7	4664		12	9381
3/5	7	9056	59/60	12	9564
	8	3448		12	9747
	8	7840		12	9930
	9	2239		13	0113
	9	6624		13	0296
	10	1016		13	0476
4/5	10	5408	119/120	13	0662
	10	9800		13	0845
	11	4192		13	1028
	11	8584	239/240	13	1211
	12	2976		13	1394
29/30	12	7368		13	1577
Sphere.	13	1760	Sphere.	13	1760

Quantity of the Moons Parallax, reſpecting the Earth at the Equator, and on every 10th Degree of Latt.

Lat.	Sphere.		Spheroid.		Diff.	
	M.	S.	M.	S.	M.	S.
10	10	40	10	35	0	5
20	20	50	20	40	0	10
30	30	25	30	10	0	15
40	39	10	38	50	0	20
50	46	25	45	55	0	30
60	52	23	51	43	0	40
70	56	35	55	45	0	50
80	59	20	58	18	1	2
90	60	00	58	41	1	19

The Quantities as the perpendiculars to the Tangents, at the ſurface of the Earth deviates from the Earths center, at every tenth Degree of Latitude.

Latitude.	Miles.	Parts.
10	15	1
20	36	3
30	54	5
40	74	6
50	74	6
60	70	6
70	50	4
80	30	2
90	00	0

N. B. At the Equator and Pole, the Plummet directs itſelf exactly to the Center, but no where elſe.

9 f